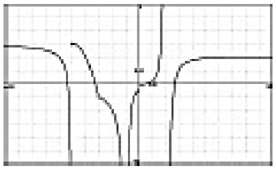
# Homework 4.2 – Horizontal and Vertical Asymptotes

**1.** (1 pt) alfredLibrary/AUCI/chapter4/lesson2/question1.pg

Determine each of the given limits for the function $f$ graphed below. Type 'inf' for $\infty$, '-inf' for $-\infty$, and 'dne' if the limit does not exist. Click on the graph to enlarge the image.



(a) $\lim_{x\to-5^-} f(x) =$ ______

(b) $\lim_{x\to-5^+} f(x) =$ ______

(c) $\lim_{x\to-5} f(x) =$ ______

(d) $\lim_{x\to-3} f(x) =$ ______

(e) $\lim_{x\to-1^-} f(x) =$ ______

(f) $\lim_{x\to-1^+} f(x) =$ ______

(g) $\lim_{x\to-1} f(x) =$ ______

(h) $\lim_{x\to2^-} f(x) =$ ______

(i) $\lim_{x\to2^+} f(x) =$ ______

(j) $\lim_{x\to2} f(x) =$ ______

(k) $\lim_{x\to\infty} f(x) =$ ______

(l) $\lim_{x\to-\infty} f(x) =$ ______

**2.** (1 pt) alfredLibrary/AUCI/chapter4/lesson2/analyzegraph2pet.pg

Let $g(x) = \frac{1}{(x+6)^3}$.

(a) Complete the table below for $x$-values close to $-6$. If a value is undefined, enter *NONE*.

| $x$ | -7 | -6.1 | -6.01 | -6 | -5.99 | -5.9 | -5 |
|---|---|---|---|---|---|---|---|
| $g(x)$ | ___ | ___ | ___ | ___ | ___ | ___ | ___ |

(b) Based on the values in the table, $g(x) \to$ ___ () as $x \to -6$ from the left.

(c) Based on the values in the table, $g(x) \to$ ___ () as $x \to -6$ from the right.

(d) Complete the two tables below to see how $g(x)$ behaves in the long-run. If a value is undefined, enter *NONE*. Enter exact answers using fractions instead of long decimal answers.

| $x$ | 10 | 100 | 1000 |
|---|---|---|---|
| $g(x)$ | ___ | ___ | ___ |

| $x$ | -10 | -100 | -1000 |
|---|---|---|---|
| $g(x)$ | ___ | ___ | ___ |

(e) Based on the values in your table, $g(x) \to$ ___ () as $x$ takes on larger and larger positive values.

(f) Based on the values in your table, $g(x) \to$ ___ () as $x$ takes on larger and larger negative values.

(g) The vertical asymptote(s) is/are $x =$ ___ ()

(h) The horizontal asymptote(s) is/are $y =$ ___. ()

**3.** (1 pt) alfredLibrary/AUCI/chapter4/lesson2/quiz/question2.pg

Analyze the behavior of the function $y = \frac{4x+32}{x^2+(-13)x+40}$ near the vertical asymptote $x = 8$. Enter 'inf' if the limit is $\infty$, enter '-inf' if the limit is $-\infty$, and enter 'dne' if the limit does not exist.

(a) $\lim_{x\to8^-} \frac{4x+32}{x^2+(-13)x+40} =$ ______

(b) $\lim_{x\to8^+} \frac{4x+32}{x^2+(-13)x+40} =$ ______

(c) $\lim_{x\to8} \frac{4x+32}{x^2+(-13)x+40} =$ ______

**4.** (1 pt) alfredLibrary/AUCI/chapter4/lesson2/analyzegraph1pet.pg

*Instructions:*

- If you are asked for a function, then enter a function.
- If you are asked to find $x$- or $y$-values, then enter either a number or a list of numbers separated by commas. If there are no solutions, enter *None* .
- If you are asked to find an interval or union of intervals, then use **interval notation**. Enter { } if an interval is empty.
- If you are asked to find a limit, then enter either a number, 'inf' for $\infty$, '-inf' for $-\infty$, or 'dne' if the limit does not exist.

Let $f(x) = \frac{5x^2}{x^2-16}$.

(a) Calculate the first derivative of $f$. (At this point, it would be wise to simplify the numerator by eliminating parentheses and combining like terms.)

$f'(x) =$ ______

(b) List all of the points where $f'(x)$ is zero or undefined (Hint: Find the zeros of the numerator and the zeros of the denominator. The points in this list that are also in the domain of $f$ are called "critical points."):

$x =$ ______________

(c) Use the points from (b) and sign tests to find the intervals on which $f$ is increasing and the intervals on which $f$ is decreasing (Hint: Your answers must exclude any points where $f'$ is undefined.):

$f$ is increasing on ______________.

$f$ is decreasing on ______________.

(d) Enter the inputs for the local extrema:

Local maximum at $x =$ ______________

Local minimum at $x =$ ______________.

(e) Find the following left- and right-hand limits at the vertical asymptote $x = -4$.

$$\lim_{x\to -4^-} \frac{5x^2}{x^2-16} = \boxed{?} \qquad \lim_{x\to -4^+} \frac{5x^2}{x^2-16} = \boxed{?}$$

(f) Find the following left- and right-hand limits at the vertical asymptote $x = 4$.

$$\lim_{x\to 4^-} \frac{5x^2}{x^2-16} = \boxed{?} \qquad \lim_{x\to 4^+} \frac{5x^2}{x^2-16} = \boxed{?}$$

(g) Find the following limits at infinity to determine any horizontal asymptotes.

$$\lim_{x\to -\infty} \frac{5x^2}{x^2-16} = \boxed{?} \qquad \lim_{x\to +\infty} \frac{5x^2}{x^2-16} = \boxed{?}$$

(h) Calculate the second derivative of $f$. (At this point, it would be wise to simplify the numerator by eliminating parentheses and combining like terms.)

$f''(x) =$ ______________

(i) List the points where the second derivative is zero or undefined:

$x =$ ________

(j) Use these points and sign tests to find the intervals of concavity:

$f$ is concave up on ______________.

$f$ is concave down on ______________

(k) Complete the following for the function $f$.

The domain of $f$ is ______________.

The $y$-intercept is ______________.

The $x$-intercepts are ______________.

(l) Sketch a graph of the function $f$ without using a graphing calculator. Plot the $y$-intercept and the $x$-intercepts, if any exist. Draw dashed lines for horizontal and vertical asymptotes. Plot the points where $f$ has local maxima, local minima, and inflection points. Use what you know about intervals of increase/decrease and concavity to sketch the remaining parts of the graph of $f$.