Activity 4.2 – Horizontal and Vertical Asymptotes

1. Since $\lim\limits_{x\to+\infty}\frac{9x^4-6x^3+2x}{2x^4+7x-1}=\lim\limits_{x\to+\infty}\frac{9x^4}{2x^4}=\lim\limits_{x\to+\infty}\frac{9}{2}=\frac{9}{2}$, the line $y=\frac{9}{2}$ is a horizontal asymptote. Similarly for $x\to-\infty$.

2. (a) $\lim\limits_{x\to+\infty}\frac{5x+x^2}{6-4x^2}=\lim\limits_{x\to+\infty}\frac{x^2}{-4x^2}=\lim\limits_{x\to+\infty}\frac{1}{-4}=-\frac{1}{4}$

   (b) $\lim\limits_{x\to-\infty}\frac{x^5-10x^2}{3x^6+x^4+1}=\lim\limits_{x\to-\infty}\frac{x^5}{3x^6}=\lim\limits_{x\to-\infty}\frac{1}{3x}=0$

   (c) $\lim\limits_{x\to+\infty}\frac{2x+x^2}{81-x}=\lim\limits_{x\to+\infty}\frac{x^2}{-x}=\lim\limits_{x\to+\infty}\frac{x}{-1}=-\infty$

   (d) $\lim\limits_{x\to-\infty}\frac{\sqrt{x^2+1}}{3x-4}=\lim\limits_{x\to-\infty}\frac{\sqrt{x^2}}{3x}=\lim\limits_{x\to-\infty}\frac{-x}{3x}=\lim\limits_{x\to-\infty}\frac{-1}{3}=-\frac{1}{3}$

3. (a) Since *f* has a vertical asymptote at $x = 2$, the limit on either side will be $+\infty$, or $-\infty$. We only need to check the signs:

   If $x\to2^-$, then $x<2$ and $(x-2)<0$. Therefore, $\lim\limits_{x\to2^-}\frac{(x+1)^2(x+2)}{(x-2)(x+3)^2}=\frac{(+)(+)}{(-)(+)}\infty=-\infty$.

   If $x\to2^+$, then $x>2$ and $(x-2)>0$. Therefore, $\lim\limits_{x\to2^+}\frac{(x+1)^2(x+2)}{(x-2)(x+3)^2}=\frac{(+)(+)}{(+)(+)}\infty=+\infty$.

   Hence, $\lim\limits_{x\to2}f(x)$ DNE.

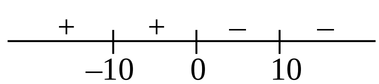
   (b) If $x\to-3^-$, then $(x+3)<0$. Therefore, $\lim\limits_{x\to-3^-}\frac{(x+1)^2(x+2)}{(x-2)(x+3)^2}=\frac{(+)(-)}{(-)(+)}\infty=+\infty$.

   If $x\to-3^+$, then $(x+3)>0$. Therefore, $\lim\limits_{x\to-3^+}\frac{(x+1)^2(x+2)}{(x-2)(x+3)^2}=\frac{(+)(-)}{(-)(+)}\infty=+\infty$.

   Hence, $\lim\limits_{x\to-3}f(x)=+\infty$.

4. (a) $f'(x)=\frac{(30x)(x^2-100)-(15x^2)(2x)}{(x^2-100)^2}=\frac{-3000x}{(x^2-100)^2}$; f' is zero at $x = 0$ and undefined at $x = 10, -10$.

   (b) Increasing on $(-\infty,-10)$ and $(-10, 0)$; decreasing on $(0, 10)$ and $(10, +\infty)$.



   (c) $\lim\limits_{x\to-10^-}\frac{15x^2}{x^2-100}=+\infty$; $\lim\limits_{x\to-10^+}\frac{15x^2}{x^2-100}=-\infty$; $\lim\limits_{x\to10^-}\frac{15x^2}{x^2-100}=-\infty$; $\lim\limits_{x\to10^+}\frac{15x^2}{x^2-100}=+\infty$

5. (a) $+\infty$; (b) $-\infty$; (c) DNE; (d) $+\infty$; (e) $+\infty$; (f) $+\infty$; (g) 1; (h) -2

6. (b) $y(2)$ must exist

   (d) $\lim\limits_{x\to2}y(x)$ must exist

   (f) $\lim\limits_{x\to2}y(x)$ must equal $y(2)$