Examples 4.1 – Analyzing Rational Functions

1. Consider the rational function $f(x) = \dfrac{x^2 - x - 2}{3x^2 - 9x + 6}$, and note that f is improper.

   Solution: Factor the numerator and denominator to get $\dfrac{x^2 - x - 2}{3x^2 - 9x + 6} = \dfrac{(x+1)(x-2)}{3(x-1)(x-2)}$.

   (i)   Domain:

   (ii) y-intercept:

   (iii) x-intercepts, holes, vertical asymptotes:

   (iv)   A proper form using long division:

2. Repeat Parts (i), (ii), and (iii) from Part 1 for the function $g(x) = \dfrac{\sqrt{x-2}}{x^2 - 9}$. (Even though g is not a rational function, a similar analysis can be done.)

   Solution: Factor the denominator to get $\dfrac{\sqrt{x-2}}{x^2 - 9} = \dfrac{\sqrt{x-2}}{(x+3)(x-3)}$.

   (i)   Domain:

   (ii) y-intercept:

   (iii) x-intercepts, holes, vertical asymptotes: