Activity 4.1 – Analyzing Rational Functions

1. (a) $N(1) = (1)^3 - 5(1)^2 + 7(1) - 3 = 0$

   (b) $g(x) = x^2 - 4x + 3$

   (c) $N(x) = (x-1)^2(x-3)$

   (d) $f(x) = \dfrac{(x-1)^2(x-3)}{(x-1)(x+1)}$

   (e) $x = 1, -1$

   (f) $x = 3$

   (b) $y = 3$

   (h) $x = -1$

   (i) $x = 1$

   (j) $f(x) = (x-5) + \dfrac{8x-8}{x^2-1}$

3. (a) $y' = \dfrac{(1)(2x-3) - (x)(2)}{(2x-3)^2} = \dfrac{-3}{(2x-3)^2}$

   (b) $y' = \dfrac{(2x+3)(x+4) - (x^2+3x)(1)}{(x+4)^2} = \dfrac{x^2+8x+12}{(x+4)^2}$

   (c) $y' = \dfrac{(2x)(3x^2-9x+6) - (x^2-2)(6x-9)}{(3x^2-9x+6)^2} = \dfrac{-9x^2+24x-18}{(3x^2-9x+6)^2}$

   (d) $y' = \dfrac{\left(\frac{1}{2\sqrt{x-2}}\cdot(1)\right)(x^2-9) - \left(\sqrt{x-2}\right)(2x)}{(x^2-9)^2} = \dfrac{-3x^2+8x-9}{2\sqrt{x-2}(x^2-9)^2}$

4. Set $y' = \dfrac{x^2+8x+12}{(x+4)^2} = \dfrac{(x+2)(x+6)}{(x+4)^2} = 0$ to get $x = -2, -6$.

5. (a) $C(3) = 0.032$ mg/cm^3

   (b) $C'(t) = \dfrac{(0.16)(t^2+6) - (0.16t)(2t)}{(t^2+6)^2} = \dfrac{-0.16t^2+0.96}{(t^2+6)^2}$ mg/cm^3 per hour

   (c) $C'(3) \approx -0.002$ mg/cm^3 per hour