Chapter 4 Review

1. **(Lesson 4.1)** Memorize the quotient rule, then practice using it.

Quotient Rule: $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

(a) If $y = \frac{9x+3}{5x+4}$, then $y' =$ ______ .

(b) If $y = \frac{6-8x+2x^2}{7-x^3}$, then $y' =$ ______ .

(c) If $y = \frac{x^3-2x+6}{7x+1}$, then $y' =$ ______ .

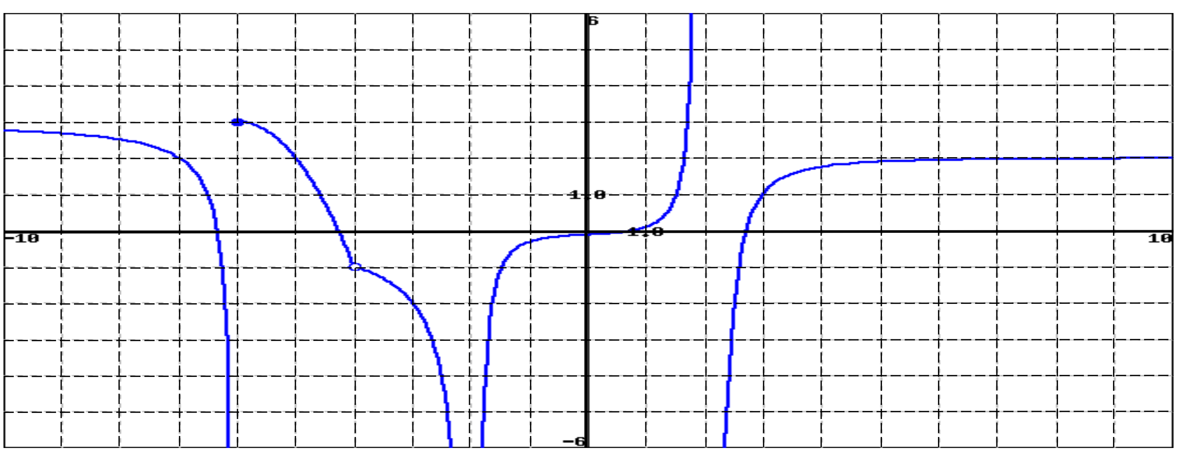
(d) If $y = \frac{-x^4}{2x^2+4x-10}$, then $y' =$ ______ .

2. **(Lesson 4.1)** The *effective population* of a species is a measurement of the average number of individuals that contribute to breeding. Let $E(x) = \frac{68x}{x+17}$ be the effective population of elephant seals containing 17 breeding males, where x is the number of breeding females.

(a) In the presence of 5 breeding females, the effective population is ______ seals.

(b) In the presence of 5 breeding females, the effective population is increasing by ______ seals per breeding female.

3. **(Lesson 4.2)** Determine each of the given limits for the function *f* graphed below.



(a) $\lim_{x \to -6^-} f(x) =$ ______

(b) $\lim_{x \to -6^+} f(x) =$ ______

(c) $\lim_{x \to -6} f(x) =$ ______

(d) $\lim_{x \to -4} f(x) =$ ______

(e) $\lim_{x \to -2^-} f(x) =$ ______

(f) $\lim_{x \to -2^+} f(x) =$ ______

(g) $\lim_{x \to -2} f(x) =$ ______

(h) $\lim_{x \to 2^-} f(x) =$ ______

(i) $\lim_{x \to 2^+} f(x) =$ ______

(j) $\lim_{x \to 2} f(x) =$ ______

(k) $\lim_{x \to +\infty} f(x) =$ ______

(l) $\lim_{x \to -\infty} f(x) =$ ______

4. **(Lesson 4.2)** Analyze the behavior of the function $y = \frac{3x+15}{x^2-13x+40}$ near the vertical asymptote $x = 5$ by evaluating the following limits.

(a) $\lim_{x \to 5^-} \frac{3x+15}{x^2-13x+40}$

(b) $\lim_{x \to 5^+} \frac{3x+15}{x^2-13x+40}$

(c) $\lim_{x \to 5} \frac{3x+15}{x^2-13x+40}$

5. **(Lesson 4.3)** Evaluate each limit. Use L'Hôpital's rule if necessary.

(a) $\lim_{x\to 4} \frac{-9x^2+38x-8}{x-4} =$ ______

(b) $\lim_{x\to -3} \frac{x^2+x-6}{(x+2)(x+3)} =$ ______

(c) $\lim_{x\to +\infty} \frac{-5x^2+3x+9}{-3x^3+2x^2-4x+8} =$ ______

(d) $\lim_{x\to +\infty} \frac{-2x^5+x^3-2x+1}{6x^5-x^3+4x} =$ ______

6. **(Lesson 4.1, 4.2, 4.3)** This problem is a complete graphical analysis of the rational function $f(x) = \frac{3x^2}{x^2-16}$.

(a) The domain (as a union of intervals) is ______.
The y-intercept is $y =$ ______.
The x-intercepts are $x =$ ______.
Vertical asymptotes at $x =$ ______.

(b) Determine the behavior of the graph near the vertical asymptotes.

$\lim_{x\to -4^-} \frac{3x^2}{x^2-16} =$ ______ $\lim_{x\to -4^+} \frac{3x^2}{x^2-16} =$ ______

$\lim_{x\to 4^-} \frac{3x^2}{x^2-16} =$ ______ $\lim_{x\to 4^+} \frac{3x^2}{x^2-16} =$ ______

(c) Find the following limits at infinity to determine any horizontal asymptotes.

$\lim_{x\to -\infty} \frac{3x^2}{x^2-16} =$ ______ $\lim_{x\to +\infty} \frac{3x^2}{x^2-16} =$ ______

(d) Calculate the first derivative of f.

(e) Find all inputs at which f' is zero or undefined. (Hint: Find the zeros of the numerator and the zeros of the denominator. The points in this list that are also in the domain of f are called "critical points.")

(f) Use the inputs from (e) and sign tests to find the intervals on which f is increasing and the intervals on which f is decreasing. Your answers must exclude any points where f' is undefined.

(g) Find the inputs for the local extrema.

(h) Calculate the second derivative of f.

(i) Find the inputs at which the second derivative is zero or undefined.

(j) Use the inputs from (i) and sign tests to find the intervals of concavity.

(k) Find the inputs for the inflection points.

(l) Sketch a graph of the function f without using a graphing calculator. Plot the y-intercept and the x-intercepts, if any exist. Draw dashed lines for horizontal and vertical asymptotes. Plot the points at which f has local maxima, local minima, and inflection points. Use what you know about intervals of increase/decrease and concavity to sketch the remaining parts of the graph of f.