Examples 3.6 – Integrals of Polynomials

1. Evaluate the integrals.

   (a) $\int \left(5x^3 - 10x^2 + 3x - 1\right)dx$ (b) $\int \left(5x^{-3} - \frac{10}{x^2} + 3\sqrt{x}\right)dx$

   Solution: (a) $\int \left(5x^3 - 10x^2 + 3x - 1\right)dx = \frac{5}{4}x^4 - \frac{10}{3}x^3 + \frac{3}{2}x^2 - x + C$

   (b) $\int \left(5x^{-3} - \frac{10}{x^2} + 3\sqrt{x}\right)dx = \int \left(5x^{-3} - 10x^{-2} + 3x^{1/2}\right)dx = -\frac{5}{2}x^{-2} + 10x^{-1} + 2x^{3/2} + C$

2. In Activity 2.6, we estimated the net area bounded by the graph of $f(x) = x^2$ on the interval [1, 3]. Use the fundamental theorem for polynomials to find the exact net area.

   Solution: By the FTC, $\int_1^3 x^2\,dx = \frac{1}{3}x^3\Big|_1^3 = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$.

3. The function $T(h) = 9.5h^3 - 15.5h^2 + 17.4h - 10.12$ gives the rate of change of the air temperature in °F per hour during the first hour-and-a-half of a thunderstorm.

   (a) Evaluate and interpret $\int_0^{1.5} T(h)dh$.

   Solution: By the FTC,

$$\begin{aligned}\int_0^{1.5} T(h)dh &= \int_0^{1.5}\left(9.5h^3 - 15.5h^2 + 17.4h - 10.12\right)dh \\ &= \left(\frac{9.5}{4}h^4 - \frac{15.5}{3}h^3 + \frac{17.4}{2}h^2 - 10.12h\right)\Big|_0^{1.5} \\ &= \left(\frac{9.5}{4}(1.5)^4 - \frac{15.5}{3}(1.5)^3 + \frac{17.4}{2}(1.5)^2 - 10.12(1.5)\right) - (0) \\ &\approx -1.02\end{aligned}$$

   This is a net decrease in temperature of about 1 °F over the first hour-and-a-half.

   (b) If the thunderstorm began at 3:00 p.m. and the temperature was 85°F, then what does the answer to Part (a) tell us about the temperature at 4:30 p.m.? At 3:30 p.m.? Explain.

   Solution: The answer to Part (a) is a *net* change, thus the temperature at 4:30 p.m. was the temperature at 3:00 p.m. plus the net change over the first 1.5 hours. Therefore,

$$\text{Temp. at 4:30 p.m.} \approx 85 + \int_0^{1.5} T(h)dh \approx 85 - 1.02 \approx 84°\text{F}$$

   Since a net change does not give information about intermediate behavior, the answer to Part (a) tells us nothing about the temperature at 3:30 p.m., but we can approximate it as

$$\text{Temp. at 3:30 p.m.} \approx 85 + \int_0^{0.5} T(h)dh$$