Activity 3.6 – Integrals of Polynomials

1. (a) $-\frac{1}{2}t^4 + \frac{5}{3}t^3 + \frac{1}{2}t^2 - 4t + C$

   (b) $\frac{9}{5}x^{5/3} + \frac{7}{x} + C$

   (c) $\frac{2}{3}x^{3/2} + C$

   (d) $\frac{3}{4}x^{4/3} + 10x^{1/2} + C$

   (e) $\int_{-1}^{1}(3u+2)^2\,dx = \int_{-1}^{1}(9u^2 + 12u + 4)dx = (3u^3 + 6u^2 + 4u)\Big|_{-1}^{1} = 14$

   (f) $\int_{1}^{4}\left(\frac{2x^3 - 32}{2x^3}\right)dx = \int_{1}^{4}\left(1 - \frac{16}{x^3}\right)dx = \left(x + \frac{8}{x^2}\right)\Big|_{1}^{4} = -\frac{9}{2}$

2. (a) $F(x) = \int(7x^{-3} - 6x^{-5})dx = -\frac{7}{2x^2} + \frac{3}{2x^4} + C$; set $F(1) = -\frac{7}{2} + \frac{3}{2} + C = 3$ to get $C = 5$;

   $F(x) = -\frac{7}{2x^2} + \frac{3}{2x^4} + 5$

   (b) Set $F(2) = -\frac{7}{8} + \frac{3}{32} + C = -\frac{41}{32}$ to get $C = -\frac{1}{2}$; $F(x) = -\frac{7}{2x^2} + \frac{3}{2x^4} - \frac{1}{2}$

3. Let $f(x) = x$ and let $g(x) = x$.

   The integral of the product is $\int f(x)\cdot g(x)\,dx = \int x^2 dx = \frac{1}{3}x^3$, but

   the product of the integrals is $\int f(x)\,dx \cdot \int g(x)\,dx = \int x dx \cdot \int x dx = \frac{1}{2}x^2 \cdot \frac{1}{2}x^2 = \frac{1}{4}x^4$.

4. (a) (i) Since $\frac{d}{dx}(k\cdot F(x)) = k\cdot f(x)$, it follows that $\int k\cdot f(x)dx = k\cdot F(x)$.

   (ii) Since $\frac{d}{dx}(F(x)) = f(x)$, it follows that $\int f(x)dx = F(x)$.

   (iii) From (ii), then (i), $k\cdot\int f(x)dx = k\cdot F(x) = \int k\cdot f(x)dx$.

   (b) (i) Since $\frac{d}{dx}(F(x) \pm G(x)) = f(x) \pm g(x)$, we have $\int (f(x) \pm g(x))dx = F(x) \pm G(x)$.

   (ii) Since $\frac{d}{dx}(F(x)) = f(x)$, we have $\int f(x)dx = F(x)$.

   (iii) Since $\frac{d}{dx}(G(x)) = g(x)$, we have $\int g(x)dx = G(x)$.

   (iv) From (ii) and (iii), then (i), $\int f(x)dx \pm \int g(x)dx = F(x) \pm G(x) = \int (f(x) \pm g(x))dx$.