Activity 3.6[‡] – Integrals of Polynomials

FOR DISCUSSION: *In your own words, state each of the following:*
The power rule for derivatives and the power rule for integrals;
The constant multiple and sum/difference rules for integrals;
The Fundamental Theorem of Calculus for polynomials.

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1. Evaluate each of the following integrals. You may need to rewrite the integrand first.

(a) $\int\left(-2t^3+5t^2+t-4\right)dt =$

(b) $\int\left(3x^{2/3}-\frac{7}{x^2}\right)dx =$

(c) $\int\sqrt{x}\,dx =$

[‡] This activity has supplemental exercises.

(d) $\int \left(\sqrt[3]{x} + \frac{5}{\sqrt{x}} \right) dx =$

(e) $\int_{-1}^{1} (3u+2)^2 \, du =$ (**HINT**: You must expand the integrand before you integrate.)

(f) $\int_{1}^{4} \left(\frac{2x^3 - 32}{2x^3} \right) dx =$ (**HINT**: You must first split the integrand into two fractions.)

2. Recall that $\int f(x)dx$ represents the infinite family of antiderivatives of f, each identified by its constant of integration, C. Given a point in the plane, we could find the constant C that identifies the unique member of the family passing through the given point.

Consider the function $f(x) = \dfrac{7}{x^3} - \dfrac{6}{x^5}$, and suppose that $F(x)$ is an antiderivative of $f(x)$.

(a) Find a formula for F such that $F(1) = 3$. That is, find the antiderivative passing through the point (1, 3)).

(b) Find a formula for F such that $F(2) = -\frac{41}{32}$. That is, find the antiderivative passing through the point $\left(2, -\frac{41}{32}\right)$.

3. (**OPTIONAL**) Just as the derivative of a product is not the product of the derivatives, the integral of a product is not the product of the integrals. That is,

$$\int (f(x)\cdot g(x))dx \neq \int f(x)dx \cdot \int g(x)dx$$

Later in this course and in Calculus II you will learn how to integrate certain types of products, but be aware that **there is not a "product rule" for integration**!

Think of two simple power functions *f* and *g* such that the integral of their product is not the product of their integrals. For simplicity, assume that all constants of integration are zero.

Let $f(x) =$ ____________ and let $g(x) =$ ____________.

The integral of the product is $\int f(x)\cdot g(x)\, dx =$ __________________, but

the product of the integrals is $\int f(x)\, dx \cdot \int g(x)\, dx =$ __________________ .

4. (**OPTIONAL**) Let's verify the general properties of the integral.

(a) **Constant Multiple Rule**: Let *F* be a function such that $F' = f$, and let *k* be a constant. For simplicity, assume that all constants of integration are zero.

(i) Since $\frac{d}{dx}\left(k\cdot F(x)\right) =$ ______________, it follows that $\int k\cdot f(x)dx =$ ______________ .

(ii) Since $\frac{d}{dx}\left(F(x)\right) =$ ______________, it follows that $\int f(x)dx =$ ______________ .

(iii) Put Parts (i) and (ii) together to deduce the constant multiple rule.

(b) **Sum/Difference Rule**: Let *F* and *G* be functions such that $F' = f$ and $G' = g$. For simplicity, assume that all constants of integration are zero.

(i) Since $\frac{d}{dx}\left(F(x) \pm G(x)\right) =$ _____________, we have $\int (f(x) \pm g(x))dx =$ _____________ .

(ii) Since $\frac{d}{dx}\left(F(x)\right) =$ __________________, we have $\int f(x)dx =$ __________________ .

(iii) Since $\frac{d}{dx}\left(G(x)\right) =$ ________________, we have $\int g(x)dx =$ __________________ .

(iv) Put Parts (i) through (iii) together to deduce the sum/difference rule.