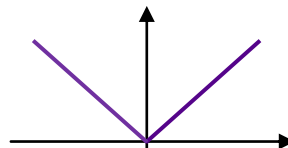




Lesson 3.5 – Piecewise Functions

A **piecewise function** is a function that is defined by different formulas along its domain. The simplest and most common piecewise function is the **absolute value function**:

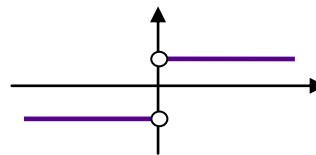
$$|x| = \sqrt{x^2} = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$



The absolute value function gives the distance from x to the origin, thus its outputs are nonnegative. The domain is the set of all real numbers, and the origin $(0, 0)$ serves as both the x - and y -intercept.

The graph of $f(x) = |x|$ is continuous (no breaks, jumps, or holes), but the graph has a sharp corner at $(0, 0)$. Equivalently, the graph has a sudden change in slope at the origin. To verify this formally, we differentiate the formulas on either side of zero (but not at zero!) to get

$$\frac{d}{dx}(|x|) = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0 \end{cases}$$



Therefore, at any point to the left of zero, the slope is -1 , and at any point to the right of zero, the slope is 1 . (We will eventually express these ideas using **left-hand limits** and **right-hand limits**:

$$\lim_{x \rightarrow 0^-} f'(x) = -1 \text{ and } \lim_{x \rightarrow 0^+} f'(x) = 1.)$$

If the slopes of a continuous function f are closing in on the same finite value from either side of a domain point $x = x_0$, then we say that f is **differentiable** at $x = x_0$ and that the derivative $f'(x_0)$ **exists**. Otherwise, the function is **not differentiable** at $x = x_0$ and the derivative **does not exist**. Algebraically, we can determine non-differentiability (and differentiability) by the domain of the derivative function. Graphically, non-differentiability occurs at a point when the following exist:

Break, jump, or hole: The graph is not continuous, so function values on either side of the point of discontinuity do not approach each other. A piecewise function may have this behavior.

Sharp corner: The graph is continuous, but slopes on either side of the sharp corner do not approach each other. A continuous piecewise function (e.g., an absolute value function) may have this behavior.

Vertical tangent: The graph is continuous and has no sharp corner, but the slopes on either side approach infinity or negative infinity. The tangent line exists, but it is vertical. An example is $f(x) = \sqrt[3]{x}$.

