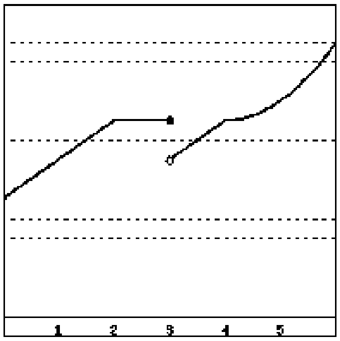
Homework 3.5 – Piecewise Functions

1. **(1 pt) alfredLibrary/AUCI/chapter3/lesson5/quiz/question5pet.pg**
Recall that a function is discontinuous at $x = a$ if the graph has a break, jump, or hole at a. Also recall that a function is non-differentiable at $x = a$ if it is not continuous at a or if the graph has a sharp corner or vertical tangent line at a.



(a) At which x-values does the function above appear to be discontinuous? $x =$ _____

(b) At which x-values does the function appear to be non-differentiable? $x =$ _____

Enter multiple answers as a comma-separated list. If there are no x-values that apply, then enter the word 'none' (without quotes).

2. **(1 pt) alfredLibrary/AUCI/chapter3/lesson5/limits5pet.pg**

Let $f(x) = \begin{cases} 2x+1, & \text{if } x \le -1 \\ x+3, & \text{if } -1 < x \le 2 \\ 2x-3, & \text{if } 2 < x \end{cases}$

Calculate the following function values:

(a) $f(-5) =$ _______

(b) $f(-1) =$ _______

(c) $f(0.5) =$ _______

(d) $f(2) =$ _______

(e) $f(5) =$ _______

3. **(1 pt) alfredLibrary/AUCI/chapter3/lesson5/limits6pet.pg**
The expression, $\lim_{x\to a} f(x)$, is called a "two-sided limit." A two-sided limit is said to "exist" if the one-sided limits are finite and equal, that is, if there is a finite number L such that

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$$

We we see later that for a function to be continuous at $x = a$, the function $f(x)$ must be defined at a, and the two-sided limit at a must exist.

Let $f(x) = \begin{cases} -x-2, & \text{if } x \le -3 \\ 2x+3, & \text{if } -3 < x \le 2 \\ x-2, & \text{if } 2 < x \end{cases}$

(a) Calculate the following limits. Enter DNE if the limit does not exist.

$\lim_{x\to -3^-} f(x) =$ _____

$\lim_{x\to -3^+} f(x) =$ _____

$\lim_{x\to -3} f(x) =$ _____

(b) Calculate the following limits. Enter DNE if the limit does not exist.

$\lim_{x\to 2^-} f(x) =$ _____

$\lim_{x\to 2^+} f(x) =$ _____

$\lim_{x\to 2} f(x) =$ _____

(c) Based on part (a), is $f(x)$ continuous at $x = -3$?

(enter "yes" or "no") ___

(d) Based on part (b), is $f(x)$ continuous at $x = 2$?

(enter "yes" or "no") ___

4. **(1 pt) alfredLibrary/AUCI/chapter3/lesson5/question3pet.pg**
For what value of the constant c is the function f continuous on the interval $(-\infty,\infty)$? (HINT: The two pieces of the graph must connect at c.)

$$f(x) = \begin{cases} x^2-7, & x \le c \\ 2x-8, & x > c \end{cases}$$

$c =$ __________

5. **(1 pt) alfredLibrary/AUCI/chapter3/lesson5/question33pet.pg**

Let $f(x) = \begin{cases} 4, & \text{if } x < -3 \\ x+4, & \text{if } -3 \le x < 3 \\ 5-x^2, & \text{if } x \ge 3 \end{cases}$

Compute $\int_{-6}^{5} f(x)dx$ by expressing it as a sum of integrals. It may be helpful to view the graph of $f(x)$.

$\int_{-6}^{5} f(x)dx =$ ____________________