# Examples 3.5 – Piecewise Functions

1. Discuss the continuity and differentiability of the function $f(x) = \begin{cases} x^2 - 6x + 6, & \text{if } x \le 2 \\ x + 1, & \text{if } x > 2 \end{cases}$.

   Solution: We must check the **left-hand limit** (a "limit from the left") and the **right-hand limit** (a "limit from the right"):

2. Discuss the continuity and differentiability of the function $g(x) = \begin{cases} 2x + 6, & \text{if } x < -2 \\ 1, & \text{if } -2 \le x < 1 \\ 2 - x, & \text{if } x \ge 1 \end{cases}$.

   Solution: We must check **one-sided limits**:
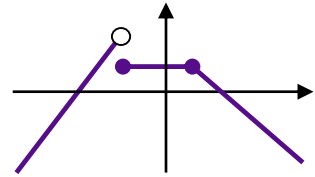
3. Find numbers a and b so that $f(x) = \begin{cases} x^2 - 6x + 6, & \text{if } x \le 2 \\ ax + b, & \text{if } x > 2 \end{cases}$ is differentiable everywhere.

   Solution: We need $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$, and $\lim_{x \to 2^-} f'(x) = \lim_{x \to 2^+} f'(x)$.