Activity 3.5 – Piecewise Functions

1. (a) $g'(x) = \begin{cases} 2x - 6, & \text{if } x < -1 \\ a, & \text{if } x > -1 \end{cases}$

   (b) Set $2(-1) - 6 = a$ to get $a = -8$; set $(-1)^2 - 6(-1) + 5 = (-8)(-1) + b$ to get $b = 4$.

2. $a = 1/2$; $b = 8$

3. (a) $f(-3) = 6$ (b) $f(-1) = -2$ (c) $f(0) = 1$

   (d) $f(1) = 2$ (e) $f(5) = -2$

4. (a) $\lim\limits_{x \to -1^-} f(x) = -2$ (b) $\lim\limits_{x \to -1^+} f(x) = 0$ (c) $\lim\limits_{x \to -1} f(x) = $ DNE

   (d) $\lim\limits_{x \to 1^-} f(x) = 2$ (e) $\lim\limits_{x \to 1^+} f(x) = 2$ (f) $\lim\limits_{x \to 1} f(x) = 2$

   (g) No   (h) Yes

5. $f'(x) = \begin{cases} 2x, & \text{if } x < -1 \\ 1, & \text{if } -1 < x < 1 \\ -1, & \text{if } x > 1 \end{cases}$

6. (a) $\lim\limits_{x \to -1^-} f'(x) = -2$ (b) $\lim\limits_{x \to -1^+} f'(x) = 1$ (c) $\lim\limits_{x \to -1} f'(x) = $ DNE

   (d) $\lim\limits_{x \to 1^-} f'(x) = 1$ (e) $\lim\limits_{x \to 1^+} f'(x) = -1$ (f) $\lim\limits_{x \to 1} f'(x) = $ DNE
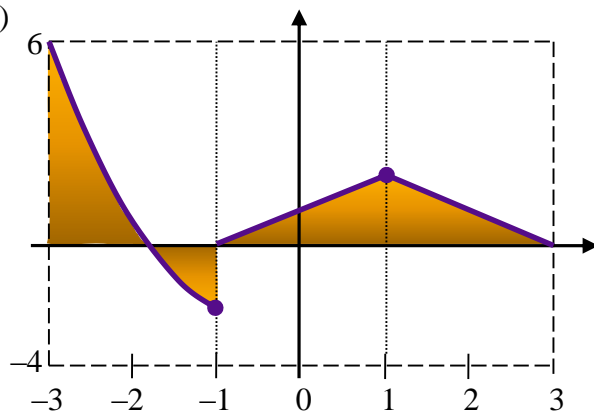
   (g) No   (h) No

7. (a) The function f is not continuous on $[-3, 3]$.

   (b)

   (c) $\int_{-3}^{3} f(x)dx = \int_{-3}^{-1}(x^2 - 3)dx + \int_{-1}^{1}(x + 1)dx + \int_{1}^{3}(3 - x)dx = \frac{8}{3} + 2 + 2 = \frac{20}{3}$