Activity 3.5* – Piecewise Functions

FOR DISCUSSION: *What is a piecewise function?*
State the definition of the absolute value function.
Graphically, what might cause a function to be non-differentiable at $x = a$?

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1. Consider the piecewise function $g(x) = \begin{cases} x^2 - 6x + 5, & \text{if } x \le -1 \\ ax + b, & \text{if } x > -1 \end{cases}$.

   (a) Compute $g'(x)$ by differentiating each "piece." (You must omit –1 from the domain.)

   (b) Find a and b so that g is differentiable at $x = -1$. (**HINT**: Use Part (a) to find a. Then use continuity to find b.)

2. Find a and b so that $h(x) = \begin{cases} -x^2 + 6x + 2, & \text{if } x \le 2 \\ ax^2 + b, & \text{if } x > 2 \end{cases}$ is differentiable at $x = 2$.

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* This activity contains new content.

You will use the piecewise function $f(x)=\begin{cases} x^2-3, & \text{if } x\le -1 \\ x+1, & \text{if } -1<x\le 1 \\ 3-x, & \text{if } x>1 \end{cases}$ for the remainder of this activity. In the TI-84, this function is entered as

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\Y1 = (X² - 3)(X ≤ -1)+(X + 1)(-1 < X)(X ≤ 1)+(3 - X)(X > 1)
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In other words, formulas and corresponding domain restrictions are multiplied together with parentheses, and then these products are added together. The inequalities can be found in the TEST menu (**[2ND]**, **[MATH]**). If your calculator is connecting the edges of the pieces together, then press **[MODE]** and switch from CONNECTED mode to DOT mode. Use your calculator to graph the function f given above in the window [–3, 3] × [–4, 6].

3. Calculate the following function values.

(a) $f(-3)=$_______ (b) $f(-1)=$_______ (c) $f(0)=$_______

(d) $f(1)=$_______ (e) $f(5)=$_______

4. The expression, $\lim_{x\to a} f(x)$, is called a "two-sided limit." The two-sided limit "exists" if the one-sided limits are finite and equal, that is, if there is a finite number L such that

$$\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)=L$$

The expression $x\to a^-$ means that x is to the left (less than) a and getting closer to a; the expression $x\to a^+$ means that x is to the right (greater than) a and getting closer to a. We will see later that for a function to be continuous at $x = a$, the function must be defined at a, and the two-sided limit at a must exist.

Compute the following limits. Write DNE if the limit does not exist. (**HINT**: Plug a into the appropriate "piece.")

(a) $\lim_{x\to -1^-} f(x)=$_______ (b) $\lim_{x\to -1^+} f(x)=$_______ (c) $\lim_{x\to -1} f(x)=$_______

(d) $\lim_{x\to 1^-} f(x)=$_______ (e) $\lim_{x\to 1^+} f(x)=$_______ (f) $\lim_{x\to 1} f(x)=$_______

(g) Based on your answer to Part (c), is f continuous at $x = -1$?

(h) Based on your answer to Part (f), is f continuous at $x = 1$?

5. Find $f'(x)$ if $f(x)=\begin{cases} x^2-3, & \text{if } x\le -1 \\ x+1, & \text{if } -1<x\le 1 \\ 3-x, & \text{if } x>1 \end{cases}$. (Omit break points from the domain!)

6. Compute the following limits. Write DNE if the limit does not exist.

(a) $\lim_{x\to -1^-} f'(x) =$ _______ (b) $\lim_{x\to -1^+} f'(x) =$ _______ (c) $\lim_{x\to -1} f'(x) =$ _______

(d) $\lim_{x\to 1^-} f'(x) =$ _______ (e) $\lim_{x\to 1^+} f'(x) =$ _______ (f) $\lim_{x\to 1} f'(x) =$ _______

(g) Based on your answer to Part (c), is f differentiable at $x = -1$?

(h) Based on your answer to Part (f), is f differentiable at $x = 1$?

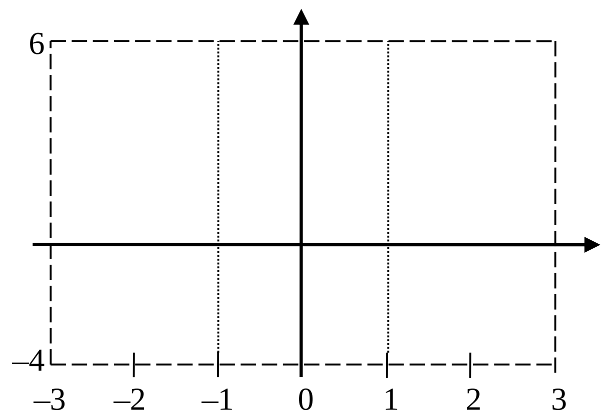
7. (**OPTIONAL**) When integrating a piecewise function, we need not worry too much about discontinuities as long as the function does not increase or decrease without bound. (You'll need to wait for Calculus II for that lesson!)

Recall, $f(x) = \begin{cases} x^2 - 3, & \text{if } x \le -1 \\ x+1, & \text{if } -1 < x \le 1 \\ 3 - x, & \text{if } x > 1 \end{cases}$.

(a) Why can't we use the Fundamental Theorem of Calculus to directly evaluate $\int_{-3}^{3} f(x)dx$?

(b) Copy the graph from your calculator onto the set of axes on the right. The domain breaks are shown to help guide you.

Shade in $\int_{-3}^{3} f(x)dx$.



(c) Set up and evaluate a sum of integrals that represents your answer to Part (b). (**HINT**: You should have three integrals.)

$\int_{-3}^{3} f(x)dx =$