Examples 3.4 – Products of Functions

1. Use the product rule to find the derivative of each of the following functions.

   (a) $y = (2x^3 - 3x)(x^2 + 5x - 10)$ (b) $y = x^2\sqrt{1 - x^3}$ (c) $y = \dfrac{x^2 - 3}{9x + 2}$

   Solutions: Note that problems (b) and (c) require both the product and chain rules.

   (a) $y = (2x^3 - 3x)(x^2 + 5x - 10) \quad \rightarrow \quad y' = (6x^2 - 3)(x^2 + 5x - 10) + (2x^3 - 3x)(2x + 5)$

   (b) $y = x^2\sqrt{1 - x^3} \quad \rightarrow \quad y' = 2x\sqrt{1 - x^3} + x^2\left(\dfrac{1}{2\sqrt{1 - x^3}} \cdot \left(-3x^2\right)\right)$

   (c) For a quotient, we can rewrite the denominator as a factor raised to the negative one power and then apply the product rule:

   $$y = \frac{x^2 - 3}{9x + 2} = (x^2 - 3) \cdot \left(9x + 2\right)^{-1} \quad \rightarrow \quad y' = (2x) \cdot \left(9x + 2\right)^{-1} + (x^2 - 3)(-1)(9x + 2)^{-2}(9)$$

2. Suppose the selling price P per unit of an item depends on x, the quantity sold. The revenue from the sale of x units at price P per unit is $R(x) = xP(x)$.

   (a) Suppose that when 500 units are sold, the price is \$5.49 per unit. Find $R(500)$.

   (b) Suppose that when 500 units are sold, the price is dropping by \$0.001 per unit. Find the rate of change of revenue when 500 units are sold.

   Solution:

   (a) We are given that $P(500) = 5.49$, so $R(500) = 500 \cdot P(500) = 500 \cdot 5.49 = 2745$. This means that the revenue from the sale of 500 items is \$2,745.

   (b) Since the revenue function is a product, its rate of change is $R'(x) = 1 \cdot P(x) + x \cdot P'(x)$. We are given $P'(500) = -0.001$, so

   $$\begin{aligned} R'(500) &= P(500) + 500 \cdot P'(500) \\ &= 5.49 + 500 \cdot (-0.001) \\ &= 4.99 \end{aligned}$$

   Therefore, when 500 units are sold, the revenue is increasing by \$4.99 per unit. As a linear approximation, this means that the sale of 501 units will yield revenue of about \$2,745 + \$4.99 = \$2,749.99.