Activity 3.4 – Products of Functions

1. (a) $f'(x) = (6)\left(x^2 + 2x - 2\right) + (6x - 7)(2x + 2)$

   (b) $g'(x) = \left(5(2x-3)^4 \cdot 2\right)\left(5x^2 + 2x\right) + (2x-3)^5(10x + 2)$

   (c) $s'(t) = \left(3t^2\right)\left(\sqrt{8t+2}\right) + \left(t^3\right)\left(\frac{1}{2\sqrt{8t+2}} \cdot 8\right)$

2. (a) $y' = 10\left(x^2 - 3x + 1\right)^9 \cdot (2x - 3)$

   $y'' = \left(90\left(x^2 - 3x + 1\right)^8 \cdot (2x-3)\right) \cdot (2x - 3) + \left(10\left(x^2 - 3x + 1\right)^9\right)(2)$

   (b) $y' = \frac{1}{3}\left(7 - 2x^2\right)^{-2/3} \cdot (-4x)$

   $y'' = \left(-\frac{2}{9}\left(7 - 2x^2\right)^{-5/3} \cdot (-4x)\right)(-4x) + \left(\frac{1}{3}\left(7 - 2x^2\right)^{-2/3}\right)(-4)$

3. $f'(x) = \left(\frac{5}{7}x^{-2/7}\right)(x-9)^2 + \left(x^{5/7}\right)(2(x-9))$

   $= x^{-2/7}(x-9)\left(\frac{5}{7}(x-9) + 2x\right)$

   $= \dfrac{(x-9)\left(\frac{19}{7}x - \frac{45}{7}\right)}{x^{2/7}}$

   Horizontal tangents at $x = 9$ and at $x = 45/19$. Vertical tangent at $x = 0$.

4. (a) $R'(t) = S'(t)P(t) + S(t)P'(t)$ dollars per day

   (b) $R'(60) = S'(60)P(60) + S(60)P'(60) = (-15)(199) + (1290)(2) = -405$ dollars per day;
   That is, the revenue was decreasing by \$405 per day.

5. (a) $(f + \Delta f)(g + \Delta g) = (f \cdot g) + (\Delta f \cdot g) + (f \cdot \Delta g) + (\Delta f \cdot \Delta g)$

   (b) $\Delta(f \cdot g) = (f + \Delta f)(g + \Delta g) - (f \cdot g) = (\Delta f \cdot g) + (f \cdot \Delta g) + (\Delta f \cdot \Delta g)$

   (c) $\dfrac{\Delta(f \cdot g)}{\Delta x} = \dfrac{(\Delta f \cdot g) + (f \cdot \Delta g) + (\Delta f \cdot \Delta g)}{\Delta x} = \dfrac{\Delta f}{\Delta x} \cdot g + f \cdot \dfrac{\Delta g}{\Delta x} + \dfrac{\Delta f}{\Delta x} \cdot \Delta g$

   (d) Since $\dfrac{df}{dx}$ exists (is finite), and $\Delta g \to 0$ as $\Delta x \to 0$, we have $\lim\limits_{\Delta x \to 0}\left(\dfrac{\Delta f}{\Delta x} \cdot \Delta g\right) = \dfrac{df}{dx} \cdot 0 = 0$.

   (e) $\dfrac{d}{dx}(f \cdot g) = \lim\limits_{\Delta x \to 0} \dfrac{\Delta(f \cdot g)}{\Delta x} = \lim\limits_{\Delta x \to 0}\left(\dfrac{\Delta f}{\Delta x} \cdot g + f \cdot \dfrac{\Delta g}{\Delta x} + \dfrac{\Delta f}{\Delta x} \cdot \Delta g\right) = \dfrac{df}{dx} \cdot g + f \cdot \dfrac{dg}{dx}$