Activity 3.4‡ – Products of Functions

FOR DISCUSSION: *Explain the product rule in your own words.*

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1. Use the product rule to find the derivative of each function.

   (a) $f(x) = (6x-7)(x^2+2x-2)$

   (b) $g(x) = (2x-3)^5(5x^2+2x)$ (**HINT**: Remember the chain rule for the first factor.)

   (c) $s(t) = t^3\sqrt{8t+2}$

2. Practice the chain and product rules by finding y' and y'' for each function.

   (a) $y = (x^2-3x+1)^{10}$

   $y' =$

   $y'' =$

‡ This activity has supplemental exercises.

(b) $y = \sqrt[3]{7-2x^2}$

$y' =$

$y'' =$

3. There are two values for x at which the graph of the function $f(x) = x^{5/7}(x-9)^2$ has a horizontal tangent line (derivative is zero), and there is one value for x at which the graph of f has a vertical tangent line (derivative is infinite). Find all three values.

   HINT: Use the product rule; note that the terms of f' have common factors $(x-9)$ and $x^{-2/7}$.

4. The revenue R from a new product at the end of day t after its release is the product of the sales S on that day and the set price P for that day. That is, $R(t) = S(t)P(t)$.

   (a) Write down a formula for the rate of change of the revenue at the end of day t with units.

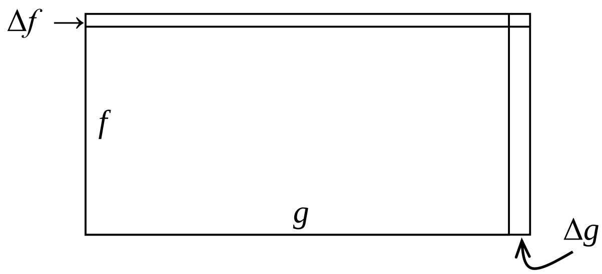
   $R'(t) =$

   (b) At the end of the 60th day, the sales were 1290 units and decreasing at a rate of 15 units per day. On the same day, the price was 199 dollars per unit and increasing at a rate of 2 dollars per unit per day. Use Part (a) to compute the rate of change of revenue at the end of day 60.

5. (**OPTIONAL**) Let f and g be functions such that the derivatives exist at x. That is, if $\Delta x \neq 0$, then

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \frac{df}{dx} \quad \text{and} \quad \lim_{\Delta x \to 0} \frac{\Delta g}{\Delta x} = \frac{dg}{dx}$$

and in particular, $\Delta g \to 0$ as $\Delta x \to 0$. Intuitively, we can think of f and g as the sides of an initial rectangle. Suppose an increase of Δx causes increases in f and g by the amounts Δf and Δg. This produces a final rectangle having sides $(f + \Delta f)$ and $(g + \Delta g)$.



(a) The area of the initial rectangle is $f \cdot g$. Find the area of the final rectangle. Your answer should have four terms.

$(f + \Delta f)(g + \Delta g) =$

(b) Find the net change in area of the two rectangles. Your answer should have three terms.

$\Delta(f \cdot g) =$

(c) Divide your answer to Part (b) by Δx to get the average rate of change in area.

$\dfrac{\Delta(f \cdot g)}{\Delta x} =$

(d) One of the terms in your answer to Part (c) should be $\dfrac{\Delta f}{\Delta x} \cdot \Delta g$. Justify why this term goes to zero as $\Delta x \to 0$.

(e) Deduce the Leibniz form of the product rule by letting $\Delta x \to 0$.