Activity 3.3[‡] – Composite Functions

FOR DISCUSSION: *What is a composite function?*
Explain the chain rule in your own words.

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1. The level L of a pollutant in the air is measured in parts per million (ppm), the population P of a community is measured in heads (h), and time t is measured in months (mo). In Parts (a) and (b), interpret the given statement in Leibniz derivative notation.

   (a) “The level of pollutant in the air is increasing by 0.06 parts per million per head.”

   (b) “The population is increasing at a rate of 4 heads per month.”

   (c) Use the chain rule to find the rate of change of the level of pollutant with respect to time (in ppm/mo).

2. Practice using the power rule with the chain rule by computing each derivative.

   (a) $y = x^{10}$ $\qquad\qquad$ $y' =$

   (b) $y = 2x^{-6}$

   (c) $y = (3x)^5$

[‡] This activity has supplemental exercises.

(d) $y = 2\left(x^3 - 7x^2\right)^4$

(e) $y = \left(x^4 + 6x^2 - 9\right)^{-2}$

(f) $y = \sqrt[5]{2x+5}$

(g) $y = \sqrt[3]{\left(1 + 2x + 3x^2\right)^4}$

(h) $y = \dfrac{5}{(1-10x)^2}$

(i) $y = \dfrac{-3}{\sqrt[5]{4x^2 + 7}}$

3. Practice using the **square root rule** with the chain rule. Use Leibnez notation for each derivative as shown.

$$y = \sqrt{3x+5} \quad \rightarrow \quad \frac{dy}{dx} = \frac{1}{2\sqrt{3x+5}} \cdot 3 = \frac{3}{2\sqrt{3x+5}}$$

(a) $y = \sqrt{5x}$

(b) $y = -5\sqrt{2x^2 - 10x}$

4. Practice using the **reciprocal rule** with the chain rule. Use Leibnez notation for each derivative as shown.

$$y = \frac{1}{3x+5} \quad \rightarrow \quad \frac{dy}{dx} = -\frac{1}{(3x+5)^2} \cdot 3 = -\frac{3}{(3x+5)^2}$$

(a) $y = \dfrac{1}{7x+2}$

(b) $y = \dfrac{7}{3x^5 - 1}$

5. (**OPTIONAL**) In chemistry, the **ideal gas law** states that the product of the pressure P (in atmospheres) and volume V (in liters) of a gas is proportional to the number of moles n in the sample times the temperature T (in Kelvin). The constant of proportionality is known as the **gas constant**, R = 0.08205 (in (L·atm)/(K·mol)). That is,

$$PV = nRT$$

Suppose a sample of gas is compressed over time t (in minutes) so that its volume is $V(t) = 6 - 0.2t$ liters. Assume that the temperature is held at a constant 215 K and the number of moles of gas is a constant 15 moles.

(a) Is the volume V increasing or decreasing over time? By how much?

(b) Find a model (with units) that expresses the pressure P as a function of time t.

(c) Find a model (with units) for the rate of change of pressure as a function of time.

(d) What is the volume after 7 minutes? What is the pressure after 7 minutes?

(e) How fast is the pressure changing after 7 minutes?