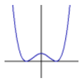

Lesson 3.2 – Polynomial Functions

A **polynomial function of degree *n*** has the form

$$y = f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

where $a_n \neq 0$, the **coefficients** are real numbers, and the exponents are nonnegative integers. The term with the highest power is the **leading term**, and the coefficient of the leading term is the **leading coefficient**. Linear, quadratic, cubic, and nonzero constant functions are all polynomials of degree one, two, three, and zero, respectively. The zero function $f(x) = 0$ has no degree. The square root function is an example of a non-polynomial function since the power on x is not a nonnegative integer.

Domain: The set of all real numbers.
Graph: Everywhere continuous and differentiable

***y*-intercept (initial value):** Set $x = 0$ and solve for y. In this case, $y(0) = f(0) = a_0$.
***x*-intercepts (roots, zeros):** Set $y = f(x) = 0$ and solve for x. This may require factoring, grouping, guess-and-check, and/or long division.

Derivative: By properties and the power rule, we can differentiate a polynomial term-by-term.

Extrema, saddle points, and intervals of increase/decrease: The graph of a polynomial function may or may not have a highest or lowest point, but it may have local extrema. In addition, the graph may or may not have a saddle point. Since the graph of a polynomial has a horizontal tangent line at each local extremum and saddle point, set $y' = f'(x) = 0$ and solve for x. These values are the critical points. Test the derivative for sign on either side of and between each critical point. A positive derivative means that the graph is increasing, and a negative derivative means that the graph is decreasing. A sign change means that the graph has an extremum, and no sign change means that the graph has a saddle point.

Inflection points: By Activity 2.4, set $y'' = f''(x) = 0$ and solve for x. Test the second derivative for sign on either side of and between each of these zeros. A sign change means the graph has an inflection point; otherwise the graph has no inflection point. (Certain non-polynomial functions may have changes in concavity at points where the first or second derivatives do not exist!)

End behavior: Refers to how a graph behaves as x tends toward infinity or negative infinity. Do function values get bigger and bigger positive, bigger and bigger negative, or do they settle on a finite number? **Limits at infinity** are used to analyze end behavior:

Limit at infinity: $\lim\limits_{x \to +\infty} f(x)$ **Limit at negative infinity:** $\lim\limits_{x \to -\infty} f(x)$

Note: For polynomials, the *leading term* dictates the end behavior.