Activity 3.2 – Polynomial Functions

1. (a) $f'(x) = -8x^7 + 10x^4 - 36x + 1$

   (b) $f''(x) = -56x^6 + 40x^3 - 36$

   (c) $f'''(x) = -336x^5 + 120x^2$

2. (a) $v(t) = 3t^2 - 20$ m/s

   (b) $a(t) = 6t$ m/s^2

   (c) $v(2) = -8$ m/s; $a(2) = 12$ m/s^2

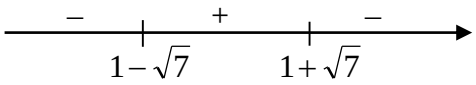
   (d) Moving left ($v < 0$); slowing down (v and a have opposite signs)

3. (a) $\lim_{x\to+\infty}\left(3x^2 - 12x^6\right) = -\infty$; $\lim_{x\to-\infty}\left(3x^2 - 12x^6\right) = -\infty$

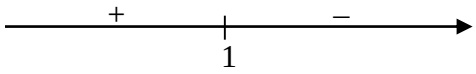
   (b) $\lim_{x\to+\infty}\left(-13x^5 + 5x^2\right) = -\infty$; $\lim_{x\to-\infty}\left(-13x^5 + 5x^2\right) = +\infty$

4. (a) Setting $-x^3 + 3x^2 + 18x = -x(x+3)(x-6) = 0$ yields $x = -3, 0, 6$.

   (b) Setting $f'(x) = -3x^2 + 6x + 18 = 0$ yields $x = 1 \pm \sqrt{7}$ (using the quadratic formula).
   Relative minimum at $x = 1 - \sqrt{7}$; relative maximum at $x = 1 + \sqrt{7}$.

   (c) Setting $f''(x) = -6x + 6 = 0$ yields $x = 1$. Inflection point at $x = 1$.

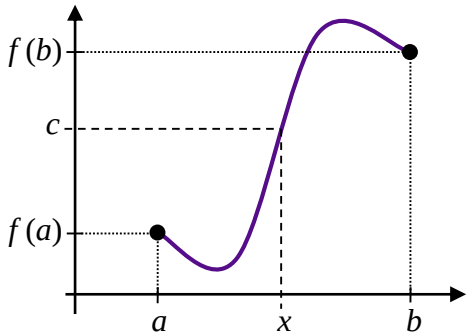
   (d) $\lim_{x\to\infty}(-x^3 + 3x^2 + 18x) = \lim_{x\to\infty}(-x^3) = -\infty$; $\lim_{x\to-\infty}(-x^3 + 3x^2 + 18x) = \lim_{x\to-\infty}(-x^3) = +\infty$

5. (b) yes;

   (c) $f(x) = c$;

   (d) continuity

   (e) Intermediate Value Theorem:
   If f is a ___continuous___ function defined on the closed interval $[a, b]$, and c is any number between ___$f(a)$___ and ___$f(b)$___, then there exists a number x between ___a___ and ___b___ such that ___$f(x) = c$___.

   (f) an x-intercept