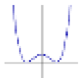

Activity 3.2[1,‡] – Polynomial Functions

FOR DISCUSSION: *What is a polynomial function?*
How do we find the extrema and inflection points (if any) of a polynomial?
What do we mean by end behavior*?*

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1. Compute each of the following for the polynomial function $f(x) = -x^8 + 2x^5 - 18x^2 + x$.

   (a) $f'(x) =$

   (b) $f''(x) =$

   (c) $f'''(x) =$

2. The equation of motion of an object in rectilinear motion is $s(t) = t^3 - 20t$, where s is in meters and t is in seconds. Assume that $t \geq 0$ and that movement to the right is positive. Each answer requires units.

   (a) The velocity v as a function of t is $v(t) =$

   (b) The acceleration a as a function of t is $a(t) =$

   (c) Find the velocity and acceleration at $t = 2$ seconds.

   $v(2) =$ $\qquad\qquad$ $a(2) =$

   (d) At $t = 2$ seconds, is the object moving to the left or to the right? Is the object speeding up or slowing down?

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[1] This activity contains new content.
[‡] This activity has supplemental exercises.

3. The end behavior of a polynomial describes its behavior as the variable approaches infinity or negative infinity. It is determined by the term with the highest power.

(a) Determine the end behavior of $f(x) = 3x^2 - 12x^6$.

$$\lim_{x \to +\infty} \left(3x^2 - 12x^6\right) = \lim_{x \to +\infty} \left(-12x^6\right)$$

$$= -12 \cdot \lim_{x \to +\infty} x^6$$

$$=$$

$$\lim_{x \to -\infty} \left(3x^2 - 12x^6\right) =$$

(b) Determine the end behavior of $g(x) = -13x^5 + 5x^2$. Set up and evaluate limits similar to Part (a).

4. Let $f(x) = -x^3 + 3x^2 + 18x$.

(a) Find the zeros (x-intercepts) of f.

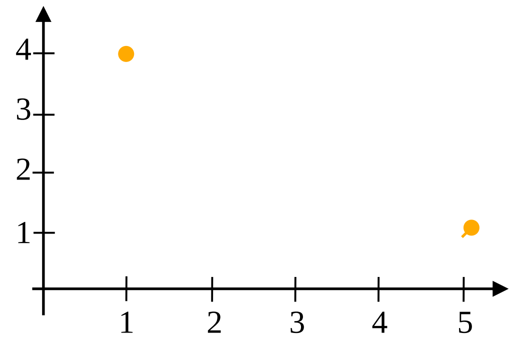
(b) Find f' and use it to find the critical points and relative extrema of f. (Show a sign test.)

(c) Find f'' and use it to find the inflection point of f. (Show a sign test.)

(d) Use limits at infinity and negative infinity to investigate the end behavior of f.

5. (**OPTIONAL**) Although we will not formally define continuity until Lesson 4.3, you should have a good understanding of it based on our experience so far. We will use our intuition about continuity to discover an important theorem called the **Intermediate Value Theorem**.

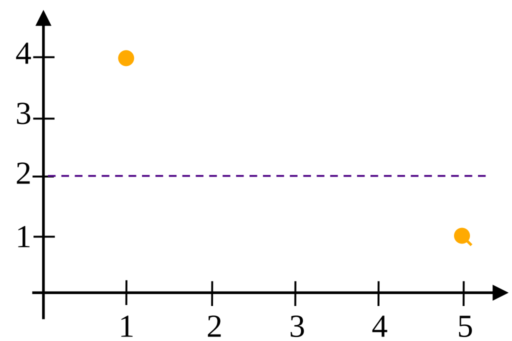
(a) The points (1, 4) and (5, 1) are shown on the given set of axes. Connect the points with a continuous function. Call it $f(x)$.



(b) Choose any number c between $f(1) = 4$ and $f(5) = 1$ on the y-axis. Sketch the horizontal line $y = c$ on the same set of axes. Does the line intersect the graph of f at least once?

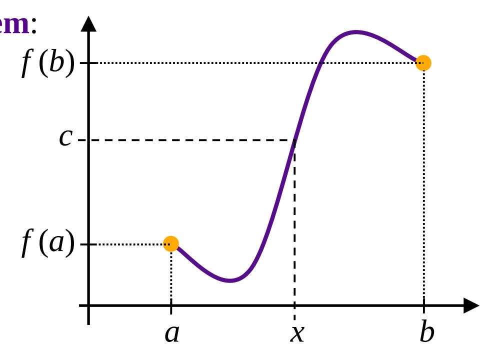
(c) Choose a point of intersection on the graphs of f and $y = c$. Mark and estimate the x-coordinate of the point you chose. Call this estimate x. What is $f(x)$?

(d) On the given set of axes, try to sketch a graph that begins at (1, 4), ends at (5, 1), but does not intersect the line $y = 2$. What property does your graph fail to have? In other words, what property would a graph require to guarantee that it intersects the horizontal line $y = c$ for <u>any</u> value c between $f(1)$ and $f(5)$?



(e) Now piece together the **Intermediate Value Theorem**:

If f is a ________________ function defined on the closed interval $[a, b]$, and c is any number between ________ and ________ , then there exists a number x between _______ and _______ such that ___________.



Note that the Intermediate Value Theorem does not tell us how to find the number x, only that at least one such number exists.

(f) Suppose f is a continuous function such that $f(2) = 3$ and $f(4) = –2$ (Make a sketch!). According to the Intermediate Value Theorem, what must f have at least one of in the interval [2, 4]?