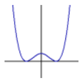

Lesson 3.1 – Power Functions

A **power function** is a function of the form $y = ax^p$, where $a, p \in \mathbf{R}$. In words, a power function is a constant multiple of the variable to a constant power.

Properties of exponents:

$$x^m x^n = x^{m+n} \qquad (x^m)^n = x^{mn} \qquad x^{-n} = \frac{1}{x^n} \qquad x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

Basic power functions: Certain power functions arise so frequently that it is a good idea for us to memorize their graphs (see Appendix A).

Type	Equation	Domain
Linear	$y = x$	$\mathbf{R}$
Quadratic	$y = x^2$	$\mathbf{R}$
Cubic	$y = x^3$	$\mathbf{R}$
Square root	$y = \sqrt{x} = x^{1/2}$	$x \geq 0$
Cube root	$y = \sqrt[3]{x} = x^{1/3}$	$\mathbf{R}$
Reciprocal	$y = \frac{1}{x} = x^{-1}$	$x \neq 0$
Reciprocal square	$y = \frac{1}{x^2} = x^{-2}$	$x \neq 0$

You may have noticed a pattern when we learned how to differentiate the first three power functions listed above, namely, "power comes down, subtract 1 from the power:"

$$\frac{d}{dx}\left(x^1\right) = 1 \cdot x^{1-1} = 1 \qquad \frac{d}{dx}\left(x^2\right) = 2 \cdot x^{2-1} = 2x \qquad \frac{d}{dx}\left(x^3\right) = 3 \cdot x^{3-1} = 3x^2$$

This pattern holds for all real number powers and is called the **power rule**. We will prove the power rule in Chapter 5.

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Power rule: $\frac{d}{dx}(x^n) = nx^{n-1}$ for any real number power n.