Examples 3.1 – Power Functions

1. Verify the power rule for $y = x^m$, where m is a positive integer.

   Solution: Observe that

$$\begin{aligned}(x-x_0)\left(x^{m-1}+x_0x^{m-2}+\cdots+x_0{}^{m-2}x+x_0{}^{m-1}\right) &= x^m + x_0x^{m-1}+\cdots+x_0{}^{m-1}x \\ &\qquad - x_0x^{m-1}-\cdots-x_0{}^{m-1}x-x_0{}^m \\ &= x^m - x_0{}^m\end{aligned}$$

   Therefore, $\dfrac{x^m - x_0{}^m}{x - x_0} = x^{m-1} + x_0x^{m-2} + \cdots + x_0{}^{m-2}x + x_0{}^{m-1}$, and it follows that

$$\frac{d}{dx}\left(x^m\right)\Big|_{x=x_0} = \lim_{x\to x_0}\frac{x^m - x_0^m}{x - x_0} = \lim_{x\to x_0}\Bigg(\underbrace{x^{m-1}+x_0x^{m-2}+\cdots+x_0{}^{m-2}x+x_0{}^{m-1}}_{(m\text{ terms})}\Bigg) = mx_0{}^{m-1}$$

2. Compute each of the following derivatives:

   Solution:

   (a) $\dfrac{d}{dx}\left(x^5\right) =$

   (b) $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) =$

   (c) $\dfrac{d}{dx}\left(\sqrt[3]{x^2}\right) =$

   (d) $\dfrac{d}{dx}\left(\dfrac{2}{x\sqrt{x}} - \dfrac{3}{\sqrt[3]{x}}\right) =$

3. According to Newton's law of universal gravitation, two masses m_1 and m_2 (in kg) at a distance of r meters apart attract each other with a force of $F(r) = G\dfrac{m_1m_2}{r^2}$ Newtons, where G is the gravitational constant. Find $F'(r)$.

   Solution: