Activity 3.1 – Power Functions

1. (a) $y' = 0$

   (b) $y' = -1$

   (c) $y' = \frac{1}{3}$

   (d) $y' = x$

   (e) $\frac{dy}{dx} = -\frac{6}{7x^4}$

   (f) $\frac{dy}{dx} = 30x^9$

   (g) $\frac{dy}{dx} = -\frac{8}{5\sqrt[5]{x}}$

   (h) $\frac{dy}{dx} = -\frac{8}{3\sqrt[3]{x^5}}$

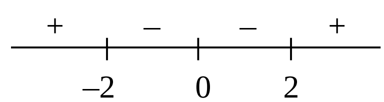
2. (a) $y' = \frac{1}{2\sqrt{x}}$

   (b) $y' = -\frac{1}{x^2}$

3. (a) $f'(r) = -\frac{8}{3r^3} - \frac{1}{2r^2}$; $f'(2) = -\frac{8}{3(2)^3} - \frac{1}{2(2)^2} = -\frac{11}{24}$

   (b) $g'(t) = \frac{25}{2}t^{3/2} - \frac{1}{3t^{2/3}}$; $g'(1) = \frac{25}{2} - \frac{1}{3} = \frac{73}{6}$

4. (a) Set $f'(x) = 3 - \frac{12}{x^2} = 0$ to get $x = 2, -2$.



   (b) local minimum at (2, 12)

   (c) local maximum at (–2, –12)

5. (a) The domain of $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$ is $x \neq 0$, so $y = \frac{1}{x}$ is not differentiable at $x = 0$.

   (b) The domain of $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$ is $x > 0$, so $y = \sqrt{x}$ is not differentiable for $x \le 0$.

6. (a)

$$\frac{d}{dx}\left(\sqrt{x}\right) = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}{\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

   (b)

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \lim_{\Delta x \to 0} \frac{\left(\frac{1}{x+\Delta x} - \frac{1}{x}\right)}{\Delta x} \cdot \frac{x(x+\Delta x)}{x(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x \cdot x(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)}$$

$$= -\frac{1}{x^2}$$