Activity 3.1[1‡] – Power Functions

FOR REVIEW: *Give three interpretations of the word* derivative.
What does the indefinite integral of a function represent?
What does the definite integral of a function on an interval represent?
Explain the FTC (for constant and linear functions) in your own words.

FOR DISCUSSION: *What is a power function? What is the power rule?*

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1. Compute the derivative of each function. Rewrite your answers without negative or fractional powers. Use prime notation for Parts (a)-(d) and Leibniz notation for Parts (e)-(h).

(a) $y = 10$

(b) $y = -x$

(c) $y = \frac{x}{3}$

(d) $y = 0.5x^2$

(e) $y = \frac{2x^{-3}}{7}$

(f) $y = 3x^{10}$

(g) $y = -2x^{4/5}$

(h) $y = \frac{4}{\sqrt[3]{x^2}}$

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[1] This activity contains new content.
[‡] This activity has supplemental exercises.

2. The square-root and reciprocal functions are very important and arise frequently. Compute their derivatives, and rewrite your answers without negative or fractional powers.

(a) $y = \sqrt{x}$ (b) $y = \dfrac{1}{x}$

NOTE: You should **memorize** these derivatives:

$$\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}} \qquad \frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

3. (a) Suppose $f(r) = \dfrac{4}{3r^2} + \dfrac{r^5}{2r^6}$. Compute $f'(r)$ and $f'(2)$.

(b) Suppose $g(t) = 5t^2\sqrt{t} - \dfrac{t}{\sqrt[3]{t^2}}$. Compute $g'(t)$ and $g'(1)$.

4. Consider the function $f(x) = 3x + \frac{12}{x}$.

(a) Compute the derivative of f and perform a number-line sign test. Note that zero is not in the domain of f, so you must mark it on your number line and test on either side of it.

(b) The function f has one local minimum. Find where the minimum occurs (the x-value), and compute the minimum value (the y-value).

(a) The function f has one local maximum. Find where the maximum occurs (the x-value), and compute the maximum value (the y-value).

5. We say that a function is **not differentiable** at x if x is not in the domain of the derivative function. In other words, if there is no well-defined slope or tangent line at x.

(a) Find all numbers x at which $y = \frac{1}{x}$ is not differentiable.

(b) Find all numbers x at which $y = \sqrt{x}$ is not differentiable.

6. (**OPTIONAL**) We will prove the power rule in Chapter 5, but for now, we can verify the derivatives of $y = \sqrt{x}$ and $y = \frac{1}{x}$ using the limit definition of the derivative function.

(a) By definition, if $y = \sqrt{x}$, then $y' = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$. Rationalize the numerator by multiplying the numerator and denominator by $\sqrt{x+\Delta x} + \sqrt{x}$ and simplifying.

$$\frac{d}{dx}\left(\sqrt{x}\right) = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}{\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$=$$

(b) By definition, if $y = \frac{1}{x}$, then $y' = \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$. Clear the fractions in the numerator by multiplying the numerator and denominator by $x(x + \Delta x)$ and simplifying.

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \lim_{\Delta x \to 0} \frac{\left(\frac{1}{x+\Delta x} - \frac{1}{x}\right)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left(\frac{1}{x+\Delta x} - \frac{1}{x}\right)}{\Delta x} \cdot \frac{x(x+\Delta x)}{x(x+\Delta x)}$$

$$=$$