

Chapter 3 Review

1. **(Lesson 3.1)** Complete the differentiation formulas.

$$\frac{d}{dx}(c) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(ax + b) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(x^n) = \underline{\hspace{2cm}} \quad \frac{d}{dx}\left(\frac{1}{x}\right) = \underline{\hspace{2cm}} \quad \frac{d}{dx}(\sqrt{x}) = \underline{\hspace{2cm}}$$

2. **(Lesson 3.1)**

(a) If $f(x) = 5 - \frac{3}{x} + \frac{1}{2x^2}$, then $f'(x) = \underline{\hspace{2cm}}$.

(b) If $g(x) = -3x^3\sqrt{x} + \frac{7}{x^2\sqrt{x}}$, then $g'(x) = \underline{\hspace{2cm}}$.

3. **(Lesson 3.2)** Let $f(x) = -6x^4 + 8x^3 + 72x^2$.

(a) The critical numbers of f are $x = \underline{\hspace{2cm}}$.

(b) The values of x for which f has a horizontal tangent line are $x = \underline{\hspace{2cm}}$.

(c) f has a relative maximum at $x = \underline{\hspace{2cm}}$.

(d) f has a relative minimum at $x = \underline{\hspace{2cm}}$.

(e) f has an inflection point at $x = \underline{\hspace{2cm}}$.

(f) $\lim_{x \rightarrow +\infty} f(x) = \underline{\hspace{2cm}}$

(g) $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$

4. **(Lesson 3.2)** The equation of motion of a particle is $s(t) = 2t^5 - 5t^2$, where s is in meters and t is in seconds. Assume that $t \geq 0$.

(a) Find the velocity v as a function of t .

(b) Find the acceleration a as a function of t .

(c) Find the acceleration after 2 seconds.

(d) Find the acceleration when the velocity is 0.

5. **(Lesson 3.3)** An environmental study in a community shows that the level of a certain pollutant in the air is $L(P) = \sqrt{P}$ parts per million (ppm), where P is the population in heads, and the population of the community is $P(t) = 70t^2 + 200$, where t is years (yr) after 2000.

(a) The population in 2010 was $\underline{\hspace{2cm}}$ people.

(b) In Leibniz notation and with units, the population in 2010 was changing by $\underline{\hspace{2cm}}$.

(c) The pollution level for 7200 people is $\underline{\hspace{2cm}}$ ppm.

(d) In Leibniz notation and with units, the pollution level for 7200 people is changing by $\underline{\hspace{2cm}}$.

(e) Write a composition function that represents the pollution level as a function of time, then use the chain rule to find its derivative with respect to time.

(f) In Leibniz notation and with units, the pollution level in 2010 was changing by $\underline{\hspace{2cm}}$.

6. Memorize the following differentiation rules, then practice using them.

Constant Multiple Rule: $(k \cdot f(x))' = k \cdot f'(x)$

Sum/Difference Rule: $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

Chain Rule: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$

Product Rule: $(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

(Lesson 3.3)

(Lesson 3.4)

(a) If $y = \sqrt[4]{9x^3 + 10}$, then $y' = \underline{\hspace{2cm}}$. (c) If $y = (3x^3 - 8x^4)(2x^5 - 1)$, then $y' = \underline{\hspace{2cm}}$.

(b) If $y = \frac{2}{(5x^3 - 10)^3}$, then $y' = \underline{\hspace{2cm}}$. (d) If $y = 2x^5(3 - 2x)^4$, then $y' = \underline{\hspace{2cm}}$.

7. **(Lesson 3.5)** Let $f(x) = \begin{cases} x^2 - 8x + 16, & \text{if } x \leq 3 \\ ax + b, & \text{if } x > 3 \end{cases}$.

Find a and b such that the function $f(x)$ is differentiable everywhere. (HINT: First use differentiability to find a . Then use continuity to find b .)

8. **(Lesson 3.6)** Memorize the following integration formulas, then practice using them.

Power Rule: If $n \neq -1$, then $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$

Constant Multiple Rule: $\int k \cdot f(x) dx = k \cdot \int f(x) dx$

Sum/Difference Rule: $\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$

(a) $\int \left(\frac{4}{x^3} - \frac{3}{x^7} + 1 \right) dx = \underline{\hspace{2cm}}$

(c) $\int_1^4 \frac{4}{\sqrt{x}} dx = \underline{\hspace{2cm}}$

(b) $\int \left(6x - 3\sqrt{x} + \frac{1}{\sqrt[5]{x^4}} \right) dx = \underline{\hspace{2cm}}$

(d) $\int_1^2 \frac{8}{x^9} dx = \underline{\hspace{2cm}}$

9. **(Lesson 3.6)** Recall that $\int f(x) dx$ represents the infinite family of antiderivatives of f , each identified by its constant of integration, C . Given a point in the plane, we could find the constant C that identifies the unique member of the family passing through the given point. Consider the function $f(x) = \frac{8}{x^3} - \frac{5}{x^7}$, and suppose $F(x)$ is the antiderivative of $f(x)$ such that $F(1) = -\frac{1}{6}$ (i.e., the graph passes through the point $(1, -\frac{1}{6})$). Then $F(x) = \underline{\hspace{2cm}}$.