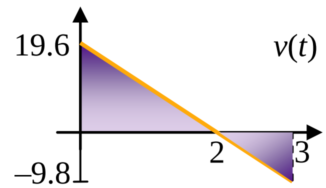
Examples 2.6 – Integrals of Linear and Quadratic Functions

1. Find three quadratic functions that have $x + 2$ as a derivative, and sketch them on the same set of axes. Then find the family of antiderivatives $\int (x+2)\,dx$.

   Solution: We want functions $ax^2 + bx + c$ such that $(ax^2 + bx + c)' = 2ax + b = x + 2$. It follows that $a = 1/2$, $b = 2$, and c is arbitrary. Three antiderivatives of $x + 2$ are $\frac{1}{2}x^2 + 2x$, $\frac{1}{2}x^2 + 2x - 1$, and $\frac{1}{2}x^2 + 2x + 3$, and the family is $\int (x+2)\,dx = \frac{1}{2}x^2 + 2x + C$.

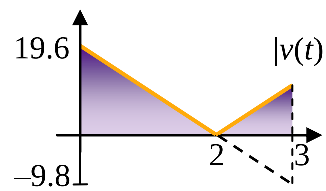
2. A projectile is fired upward from a 15.3 m cliff and allowed to fall into a valley below. The velocity of the projectile at time t is given by $v(t) = -9.8t + 19.6$ m/s. Find the displacement and total distance traveled on [0, 3] using integration. Compare with Example 1.5.2(b).

   Solution: The displacement on the interval [0, 3] is represented geometrically by the *net* area bounded by the graph of $v(t)$ on [0, 3]. This quantity is represented symbolically as the definite integral of velocity on [0, 3]. By the Fundamental Theorem,



$$\int_0^3 (-9.8t + 19.6)\,dt = (-4.9t^2 + 19.6t)\Big|_0^3 = (-4.9(3)^2 + 19.6(3)) - (-4.9(0)^2 + 19.6(0)) = 14.7 \text{ m}.$$

   The total distance traveled on [0, 3] is represented geometrically by the *total* area bounded by the graph of $v(t)$ on [0, 3]. But this is the same as the net area bounded by the *absolute value* of $v(t)$. (**[2ND] [0] [ENTER]** or **[MATH] [▶] [ENTER]** on TI-84.) Note that we can integrate v as usual on [0, 2], but since the absolute value turns negatives to positives, we must integrate $-v(t) = 9.8t - 19.6$ on [2, 3]. Therefore, the total distance traveled is



$$\int_0^3 |-9.8t + 19.6|\,dt = \int_0^2 (-9.8t + 19.6)\,dt + \int_2^3 (9.8t - 19.6)\,dt = \left(-4.9t^2 + 19.6t\right)\Big|_0^2 + \left(4.9t^2 - 19.6t\right)\Big|_2^3$$
$$= \cdots = 19.6 + 4.9 = 24.5 \text{ meters}.$$

3. If $h'(t) = 4.2t$ is the rate of increase of the number of acres consumed by a forest fire per day, then what does $\int_2^7 h'(t)\,dt$ represent, and what are its units? Calculate the integral.

   Solution: Since $h'(t) = 4.2t$ is positive on [2, 7], the integral $\int_2^7 h'(t)\,dt$ represents the total number of acres consumed between the second and seventh days. In this case,

$$\int_2^7 (4.2t)\,dt = (2.1t^2)\Big|_2^7 = (2.1(7)^2) - (2.1(2)^2) = 94.5 \text{ acres}.$$