



Activity 2.5 – Linear Approximation

1. (a) $\frac{dy}{dx} = -6x^2 + 2x$

$$\frac{d^2y}{dx^2} = -12x + 2$$

(b) $\left. \frac{dy}{dx} \right|_{x=-1} = (-6x^2 + 2x) \Big|_{x=-1} = -6(-1)^2 + 2(-1) = -8$

$$\left. \frac{d^2y}{dx^2} \right|_{x=-1} = (-12x + 2) \Big|_{x=-1} = -12(-1) + 2 = 14$$

2. (a) $\Delta y = y(2.03) - y(2) = 2(2.03)^3 - 2(2)^3 = 0.731$

(b) $dy = 6x^2 dx = 6(2)^2(.03) = 0.720$

3. (a) $f(2) = 4$; $f'(2) = 12$

(b) $y - 4 = 12(x - 2)$; $y = 12x - 20$

(c) $f(2.01) \approx 12(2.01) - 20 = 4.12$

(d) $f(2.01) = 2(2.01)^3 - 3(2.01)^2 = 4.120902$

4. (a) $dA = 10x dx = 10(36)(\pm 0.125) = \pm 45 \text{ in}^2$

(b) $\frac{dA}{A} = \frac{10x dx}{5x^2} = \frac{10(36)(\pm 0.125)}{5(36)^2} \approx \pm 0.0069 = \pm 0.69\%$

5. $dV = 4\pi r^2 dr = 4\pi(19)^2(\pm 0.5) \approx \pm 2268.23 \text{ cm}^3$

$$\frac{dV}{V} = \frac{4\pi r^2 dr}{(4/3)\pi r^3} = \frac{4\pi(19)^2(\pm 0.5)}{(4/3)\pi(19)^3} \approx \pm 0.0789 = \pm 7.89\%$$

6. $dA = 2\pi r dr = 2\pi(24)(\pm 0.1) \approx \pm 15.08 \text{ cm}^2$

$$\frac{dA}{A} = \frac{2\pi r dr}{\pi r^2} = \frac{2\pi(24)(\pm 0.1)}{\pi(24)^2} \approx \pm 0.0083 = \pm 0.83\%$$

7. $dV = 74\pi r dr = 74\pi(12)(\pm 0.15) \approx \pm 418.46 \text{ m}^3$

$$\frac{dV}{V} = \frac{74\pi r dr}{37\pi r^2} = \frac{74\pi(12)(\pm 0.15)}{37\pi(12)^2} = \pm 0.025 = \pm 2.5\%$$