Activity 2.5[‡] – Linear Approximation

FOR DISCUSSION: *What is Leibniz notation and why is it useful?*
Describe the geometrical basis behind linear approximation.
Explain how linear approximation is used in analyzing error propagation.

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1. (a) Using Leibniz notation, find the first and second derivatives of $y = -2x^3 + x^2 - 1$.

   (b) Using Leibniz notation, evaluate the derivatives in Part (a) at $x = -1$.

2. Recall that $\Delta y = y(x + \Delta x) - y(x)$ and $dy = y'(x)dx$. Let $y = 2x^3$.

   (a) Find Δy when $x = 2$ and $\Delta x = 0.03$.

   (b) Find dy when $x = 2$ and $\Delta x = 0.03$. (Note that dy is an approximation of Δy.)

[‡] This activity has supplemental exercises.

3. Let $f(x) = 2x^3 - 3x^2$.

(a) Compute $f(2)$ and $f'(2)$.

(b) Write down a point-slope form of the equation for the tangent line to the graph of f at $x = 2$. Then write the tangent line in slope-intercept form.

(c) Use the tangent line at $x = 2$ from Part (b) to approximate $f(2.01)$.

$f(2.01) \approx$

(d) Use your calculator to find the exact value of $f(2.01)$ and compare it to your approximation in Part (c). Is your approximation a good one?

$f(2.01) =$

4. A cubic wooden box with no top has four sides and a bottom, each having a surface area of x^2 square inches. Therefore, the surface area of the box is $A = 5x^2$ square inches. An edge is measured as $x = 36$ inches, with possible error in measurement of $\Delta x = \pm 0.125$ inches. The measurement is then used to calculate the surface area of the box.

(a) Use dA to approximate the propagated error ΔA in the surface area calculation.

(b) Use dA/A to approximate the relative error $\Delta A/A$ in the surface area calculation.

5. The radius of a sphere was measured to be 19 cm with a maximum possible error of ±0.5 cm. Use a linear approximation to estimate the propagated and relative errors in the calculated volume. **(HINT**: The volume of a sphere of radius r is $V = (4/3)\pi r^3$.)

6. The radius of a circular disk is measured as 24 cm with a maximum error in measurement of ±0.1 cm. Estimate the propagated and relative errors in the calculated area of the disk.

7. An oil tank in the form of a right circular cylinder of radius r has a height h of 37 meters and a volume of $V = 37\pi r^2$. The radius is measured as 12 meters with a maximum possible error of ±0.15 meters. Estimate the propagated and relative errors in the calculated volume.