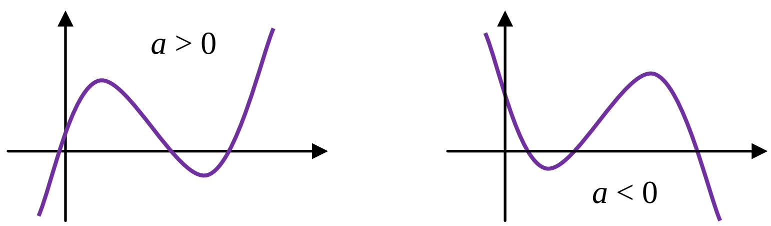
Lesson 2.4 – Analyzing Cubic Functions

A **cubic function** is a function of the form $y = f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, $d \in \mathbf{R}$, $a \neq 0$

Domain: The set of all real numbers.

Graph: Everywhere continuous (no breaks, jumps, holes) and no corners or vertical tangents (differentiable).

Orientation: The graph "starts low and ends high" if $a > 0$, and the graph "starts high and ends low" if $a < 0$.



***y*-intercept (initial value):** Set $x = 0$ and solve for y. In this case, $y(0) = f(0) = d$.

***x*-intercepts (roots, zeros):** Set $y = f(x) = 0$ and solve for x. This may require factoring, grouping, guess-and-check, long division, or technology.

Derivative: By Activity 2.3, if $f(x) = ax^3 + bx^2 + cx + d$, then $f'(x) = 3ax^2 + 2bx + c$.

Extrema, saddle points, and intervals of increase/decrease: The graph of a cubic function has no highest point and no lowest point, but it may have a "local" maximum and a "local" minimum. These points are collectively called **local extrema**. If not, then the graph may have a <u>saddle point</u>, which is a point where the graph levels off instantaneously and then continues on in the same direction. Since the graph has a **horizontal tangent line** at each extremum and saddle point, set $y' = f'(x) = 0$ and solve for x. These values are called **critical points**. Test the derivative for sign on either side of and between each critical point. A positive derivative means that the graph is increasing, and a negative derivative means that the graph is decreasing. A sign change means that the graph has an extremum, and no sign change means that the graph has a saddle point.

Special forms: **Difference of cubes:** $x^3 - a^3$ Factors as $(x - a)(x^2 + ax + a^2)$. The only x-intercept is $x = a$.

Sum of cubes: $x^3 + a^3$ Factors as $(x + a)(x^2 - ax + a^2)$. The only x-intercept is $x = -a$.