Activity 2.4 – Analyzing Cubic Functions

1. (a) The graph is…………………….. Increasing [Decreasing]
      The derivative (slope) is………… Positive [Negative]
      The derivative (slope) is………… [Increasing] Decreasing

   (b) The graph is…………………….. [Increasing] Decreasing
      The derivative is……………….. [Positive] Negative
      The derivative is…………………. [Increasing] Decreasing

2. (a) The graph is…………………….. [Increasing] Decreasing
      The derivative is…………………. [Positive] Negative
      The derivative is…………………. Increasing [Decreasing]

   (b) The graph is…………………….. Increasing [Decreasing]
      The derivative is……………….. Positive [Negative]
      The derivative is…………………. Increasing [Decreasing]

3. $y' = 3x^2 - 4x - 5$; $y'' = 6x - 4 = 0$ yields $x = 2/3$. A sign test shows that y is concave up on $(2/3, \infty)$ and concave down on $(-\infty, 2/3)$. The inflection point is at $x = 2/3$ and the coordinates are $(2/3, 56/27)$.

4. (a) $s'(t) = 3t^2 - 18t + 27 = 0$ yields $t = 3$.

   $\begin{array}{ccc} + & & + \\ \hline & 3 & \end{array}$ s'

   $s''(t) = 6t - 18 = 0$ yields $t = 3$.

   $\begin{array}{ccc} - & & + \\ \hline & 3 & \end{array}$ s''

   (b) Speeding up on $(3, \infty)$; slowing down on $(-\infty, 3)$.

5. (a) $(x - 5)(x^2 + 5x + 25)$; x-intercept at $x = 5$.

   (b) $(x + 4)(x^2 - 4x + 16)$; x-intercept at $x = -4$.

6. $x^3 + 2x^2 - 5x - 6 = (x + 1)(x^2 + x - 6) = (x + 1)(x + 3)(x - 2)$; the solutions are $x = -1, -3, 2$.

7. $x^3 - 5x^2 - 12x + 60 = x^2(x - 5) - 12(x - 5) = (x - 5)(x^2 - 12) = (x - 5)\left(x + \sqrt{12}\right)\left(x - \sqrt{12}\right)$; the solutions are $x = 5, \sqrt{12}, -\sqrt{12}$.