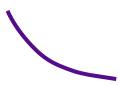
Activity 2.4*† – Analyzing Cubic Functions

FOR DISCUSSION: *Informally, what is a continuous function? A differentiable function? Explain what extrema and saddle points are and how to find them. What property does a cubic graph have that a quadratic graph does not?*

You may have noticed that the graph of a cubic function has a property that linear and quadratic graphs do not, namely a change in curvature, or a change in **concavity**. Recall, a function is increasing on an interval if and only if its derivative is positive on that interval.

1. The following two graphs are **concave up**. Circle the correct answer in each part.

(a) 

The graph is……………………….	Increasing	Decreasing
The derivative (slope) is……..…..	Positive	Negative
The derivative (slope) is………..	Increasing	Decreasing

(b) 

The graph is……………………….	Increasing	Decreasing
The derivative (slope) is…..……...	Positive	Negative
The derivative (slope) is………..	Increasing	Decreasing

Note that in the case of **concave up**, the derivative function is increasing (not necessarily the graph itself). If the derivative function is increasing, then the *derivative* of the derivative function must be positive. That is, the second derivative must be positive.

2. The following two graphs are **concave down**. Circle the correct answer in each part.

(a) 

The graph is……………………..	Increasing	Decreasing
The derivative is………………...	Positive	Negative
The derivative is…………………	Increasing	Decreasing

(b) 

The graph is……………………..	Increasing	Decreasing
The derivative is…………….……..	Positive	Negative
The derivative is…………………	Increasing	Decreasing

Note that in the case of **concave down**, the derivative function is decreasing (not necessarily the graph itself). If the derivative function is decreasing, then the *derivative* of the derivative function must be negative. That is, the second derivative must be negative.

A point at which concavity changes from concave up to concave down, or from concave down to concave up, is called an **inflection point**. To find these, look for a sign change in your sign test.

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* This activity contains new content.

† This activity is referenced in Lesson 3.2.

To find intervals of concavity and inflection points for f: Set f'' equal to zero, solve for x, plot the solutions on a number line, and perform a sign test for f'':

If f'' is positive on an interval, then f is concave up on that interval.
If f'' is negative on an interval, then f is concave down on that interval.
If f'' changes sign at a zero of f'', then f has an inflection point at that value.

3. Find the intervals of concavity and the inflection points of $y = x^3 - 2x^2 - 5x + 6$.

4. Let $s(t) = t^3 - 9t^2 + 27t - 243$ be the position function of an object in rectilinear motion.

(a) Find the first and second derivatives (velocity and acceleration). For each derivative, find the zeros and perform a number-line sign test.

(b) Recall, an object is speeding up when the velocity and acceleration have the same sign, and slowing down when they have different signs. Use Part (a) to determine the intervals on which the object is speeding up and slowing down.

SUMMARY: Given $y = f(x)$.

To find:		
	x-intercepts of f …	set $y = 0$ and solve for x
	intervals of positive/negative…	perform a sign test for y
	extrema/saddle of f …	set $y' = 0$ and solve for x (sign change?)
	intervals of increase/decrease…	perform a sign test for y'
	inflection points…	set $y'' = 0$ and solve for x (sign change?)
	intervals of concavity…	perform a sign test for y''

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5. (a) Factor the difference of cubes, $x^3 - 125$. What are the x-intercepts of $f(x) = x^3 - 125$?

   (b) Factor the sum of cubes, $x^3 + 64$. What are the x-intercepts of $f(x) = x^3 + 64$?

6. Note that $x = -1$ is a solution to the equation $x^3 + 2x^2 - 5x - 6 = 0$. That is, $(x + 1)$ is a factor of $x^3 + 2x^2 - 5x - 6$. Use long division to find any remaining solutions.

7. Find the solutions to the equation $x^3 - 5x^2 - 12x + 60 = 0$ by grouping.