Examples 2.3 – Definition and Properties of the Derivative

1. Use the "Δx" version of the limit definition of the derivative at a point to find the slope of the curve $f(x) = 3x^2 - 1$ at $x = 2$. Then use the "Δx" version of the limit definition of the derivative function to find the slope formula.

   Solution: The slope at $x = 2$ is

$$\begin{aligned} f'(2) &= \lim_{\Delta x \to 0} \frac{f(2+\Delta x) - f(2)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{[3(2+\Delta x)^2 - 1] - [3(2)^2 - 1]}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{[12 + 12\Delta x + 3\Delta x^2 - 1] - [12 - 1]}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{12\Delta x + 3\Delta x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} (12 + 3\Delta x) \\ &= 12 \end{aligned}$$

   The slope formula is

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{[3(x+\Delta x)^2 - 1] - [3(x)^2 - 1]}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{[3x^2 + 6x\Delta x + 3\Delta x^2 - 1] - [3x^2 - 1]}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{6x\Delta x + 3\Delta x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} (6x + 3\Delta x) \\ &= 6x \end{aligned}$$

2. In Lesson 2.1, we used a "three-step method" to get $(ax^2 + bx + c)' = 2ax + b$. Derive this formula using the properties of derivatives.

   Solution: We have

$$\begin{aligned} \left(ax^2 + bx + c\right)' &= \left(ax^2\right)' + \left(bx\right)' + \left(c\right)' && \text{(sum/diff rule)} \\ &= a\left(x^2\right)' + b\left(x\right)' + \left(c\right)' && \text{(const. mult. rule)} \\ &= a(2x) + b(1) + (0) \\ &= 2ax + b \end{aligned}$$

3. Use the method in Part 2 to find the derivative of $g(x) = 9x^2 - 14x + 7$.

   Solution: By the properties of the derivative, $g'(x) = \left(9x^2\right)' - \left(14x\right)' + \left(7\right)' = 18x - 14$.