Activity 2.3 – Definition and Properties of the Derivative

1. (a) $$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{\left(9(x+\Delta x)+2\right)-\left(9x+2\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\left(9x+9\Delta x+2\right)-\left(9x+2\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{9x+9\Delta x+2-9x-2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{9\Delta x}{\Delta x} \\ &= \lim_{\Delta x \to 0} 9 \\ &= 9 \end{aligned}$$

   (b) $f'(x) = 4x$

   (c) $f'(x) = 6x - 2$

2. $$\begin{aligned} y' &= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2 \cdot \Delta x + 3x \cdot \Delta x^2 + \Delta x^3 - x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{3x^2 \cdot \Delta x + 3x \cdot \Delta x^2 + \Delta x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} (3x^2 + 3x \cdot \Delta x + \Delta x^2) \\ &= 3x^2 \end{aligned}$$

3. (a) $f'(x) = \left(ax^3 + bx^2 + cx + d\right)' = \left(ax^3\right)' + \left(bx^2\right)' + \left(cx\right)' + \left(d\right)' = 3ax^2 + 2bx + c$

   (b) (i) $f'(x) = 6x^2 - 2x$; $f''(x) = 12x - 2$

   (ii) $g'(x) = -3.6x^2 + 4.6x + 0.6$; $g''(x) = -7.2x + 4.6$

4. $$\begin{aligned} \left(k \cdot f(x)\right)' &= \lim_{\Delta x \to 0} \frac{k \cdot f(x+\Delta x) - k \cdot f(x)}{\Delta x} \\ &= k \cdot \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= k \cdot f'(x) \end{aligned}$$

$$\begin{aligned} \left(f(x)+g(x)\right)' &= \lim_{\Delta x \to 0} \frac{\left[f(x+\Delta x)+g(x+\Delta x)\right]-\left[f(x)+g(x)\right]}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)+g(x+\Delta x)-g(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x)-f(x)}{\Delta x} + \frac{g(x+\Delta x)-g(x)}{\Delta x}\right] \\ &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} + \lim_{\Delta x \to 0} \frac{g(x+\Delta x)-g(x)}{\Delta x} \\ &= f'(x) + g'(x) \end{aligned}$$