Activity 2.3[1]†‡ – Definition and Properties of the Derivative

FOR DISCUSSION: *Explain the significance of each symbol:* $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

Explain in words the constant multiple and sum/difference rules.

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1. Use the limit definition of the derivative to find the derivative of each function. Show all work and use correct notation. Check each answer by computing the derivative using the differentiation formulas.

   (a) $f(x) = 9x + 2$

   $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left(9(x+\Delta x)+2\right) - \left(9x+2\right)}{\Delta x}$$

   $$=$$

   (b) $f(x) = 2x^2 - 7$

   $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left(2(x+\Delta x)^2 - 7\right) - \left(2x^2 - 7\right)}{\Delta x}$$

   $$=$$

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[1] This activity contains new content.

† This activity is referenced in Lessons 2.3 and 2.4.

‡ This activity has supplemental exercises.

(c) $f(x) = 3x^2 - 2x - 1$ (Use the limit definition!)

$f'(x) =$

2. In Lesson 2.4, our focus will be on *cubic functions* of the form $f(x) = ax^3 + bx^2 + cx + d$. By the sum/difference rule, we can differentiate f term by term, but the first term involves $y = x^3$, a function for which we do not have a derivative. Use the limit definition to find a formula for the derivative of $y = x^3$. (HINT: $(x + \Delta x)^3 = x^3 + \left(3x^2 \cdot \Delta x\right) + \left(3x \cdot \Delta x^2\right) + \Delta x^3$.)

$$y' = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x} =$$

3. (a) Use Problem 2 and the properties of the derivative to find a formula for the derivative of $f(x) = ax^3 + bx^2 + cx + d$. (**HINT**: This problem is similar to Example 2.3.2.)

$$f'(x) = \left(ax^3 + bx^2 + cx + d\right)' =$$

(b) Find the first and second derivatives of each function term-by-term.

(i) $f(x) = 2x^3 - x^2 + 10$

$f'(x) =$

$f''(x) =$

(ii) $g(x) = -1.2x^3 + 2.3x^2 + 0.6x - 10$

$g'(x) =$

$g''(x) =$

4. (**OPTIONAL**) Use the definition of the derivative to prove the constant multiple and sum/difference rules. Assume that a constant multiple can be factored out of a limit and that the limit of a sum is the sum of the limits.

Constant Multiple Rule:

$$(k \cdot f(x))' = \lim_{\Delta x \to 0} \frac{k \cdot f(x + \Delta x) - k \cdot f(x)}{\Delta x} =$$

Sum/Difference Rule:

$$(f(x) + g(x))' = \lim_{\Delta x \to 0} \frac{[f(x + \Delta x) + g(x + \Delta x)] - [f(x) + g(x)]}{\Delta x}$$

$$=$$