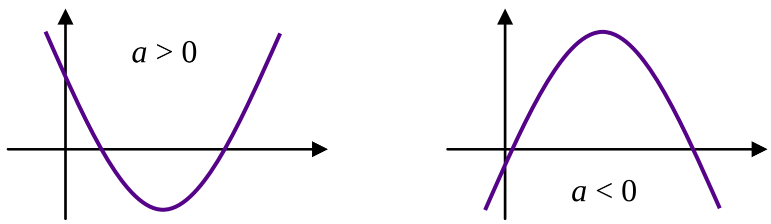
Lesson 2.2 – Analyzing Quadratic Functions

Quadratic function: A function of the form $y = f(x) = ax^2 + bx + c$, where $a, b, c \in \mathbf{R}$, $a \neq 0$

Domain: The set of all real numbers.
Graph: A parabola, which is continuous and has no sharp corners or vertical tangents. (A graph having these properties is called **differentiable**. We will formally define this later.)
Orientation: Graph opens upward if $a > 0$ and opens downward if $a < 0$.



y-intercept (initial value): Set $x = 0$ and solve for y. In this case, $y(0) = f(0) = c$.
x-intercepts (roots, zeros): Set $y = f(x) = 0$ and solve for x. This may require factoring or using the quadratic formula:

Quadratic Formula: If $f(x) = ax^2 + bx + c$, then the roots (x-int's) are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The expression $b^2 - 4ac$ is called the **discriminant**, and it tells about the roots:

$b^2 - 4ac > 0 \quad \rightarrow \quad$ 2 distinct real roots
$b^2 - 4ac = 0 \quad \rightarrow \quad$ 1 repeated real root
$b^2 - 4ac < 0 \quad \rightarrow \quad$ 0 real roots (2 complex roots)

Vertex: The vertex of a parabola is the lowest point (minimum) if $a > 0$, or the highest point (maximum) if $a < 0$. Since the graph has a **horizontal tangent line** at the vertex, set $y' = f'(x) = 0$ and solve for x to find the x-coordinate of the vertex.

Intervals of increase/decrease: Most directly deduced from the sign of a and the vertex. Otherwise, test the derivative for sign on both sides of the vertex. A positive derivative means that the graph is increasing, and a negative derivative means that the graph is decreasing.

Special forms: **Difference of squares:** $x^2 - a^2$ — Factors as $(x+a)(x-a)$. The x-intercepts are $x = \pm a$.

Sum of squares: $x^2 + a^2$ — Never factors over the reals. Has no x-intercepts.