# Examples 2.2 – Analyzing Quadratic Functions

1. Let $T(t) = -0.8t^2 + 2t + 79$ be the air temperature in degrees Fahrenheit between 6 A.M. and 8 P.M., where $t$ is hours after noon (see Examples 1.1).

   (a) Find the time at which the high temperature occurred. Find the high temperature.

   **Solution:** The high temperature occurred at the vertex of the graph of $T$. Since the graph has a horizontal tangent line at the vertex, we set $T'(t) = -1.6t + 2 = 0$ and find that $t = 1.25$ hours after noon. Note that 1.25 hours is equal to 1 hour and 15 minutes. Therefore, the high temperature occurred at 1:15 P.M. and was $T(1.25) = 80.25$ °F.

   (b) Find the temperature at 4:00 P.M. and determine how quickly it was changing at that time.

   **Solution:** We have $T(4) = 74.2$ and $T'(4) = -4.4$. Therefore, the temperature at 4 P.M. was 74.2 °F, and the temperature was falling at a rate of 4.4 °F/hr. (In Examples 1.1.4, we approximated that the temperature at 4 P.M. was 71 °F.)

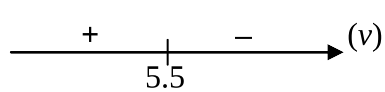
2. Let $s(t) = -6t^2 + 66t - 108$ be the position of an object in rectilinear motion, where $s$ is in meters and $t$ is in seconds. Assume that the positive direction is to the right.

   (a) Find the initial position ($y$-intercept), and determine the time(s) at which the object is at the origin ($t$-intercepts).

   **Solution:** Since $s(0) = -108$, the initial position is 108 m to the left of the origin. To find the $t$-intercepts, we set $-6t^2 + 66t - 108 = 0$ and solve for $t$. We could factor in this case to get $-6t^2 + 66t - 108 = -6(t^2 - 11t + 18) = -6(t - 2)(t - 9) = 0$. Therefore, the object is at the origin at $t = 2$ seconds and at $t = 9$ seconds.

   (b) Find the velocity function and use it to find the initial velocity, the time(s) at which the object is at rest, and the time intervals on which the object is moving to the left and right.

   **Solution:** The velocity $v(t) = s'(t) = -12t + 66$ m/s. Therefore, the initial velocity is $v(0) = 66$ m/s, and the object is at rest when $v(t) = -12t + 66 = 0$, or at $t = 5.5$ seconds. We check the sign of the derivative on either side of $t = 5.5$. Choosing $t = 4$ and $t = 6$ gives $v(4) = 18 > 0$ and $v(6) = -12 < 0$. Hence, the object is moving to the right on the interval $(-\infty, 5.5)$, and moving to the left on the interval $(5.5, +\infty)$.




   (c) Find the acceleration function.

   **Solution:** The acceleration is $a(t) = v'(t) = -12$ m/s$^2$, which is causing a constant "pull" in the negative direction.