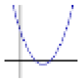

Activity 2.2[†] – Analyzing Quadratic Functions

FOR DISCUSSION: *Informally, what is a "differentiable" function?*
How can the derivative tell us about intervals of increase or decrease?
How can we use the derivative to find the vertex of a quadratic function?

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1. Solve each equation for *x*. You may need to factor and/or use the quadratic formula.

   (a) $x^2 - 16 = 0$

   (b) $3x^3 - 3x = 0$

   (c) $\frac{1}{2}x^4 + 2x^2 = 0$

   (d) $5x^2 = 9x - 4$ (**HINT**: Rewrite the equation with all terms on the left-hand side.)

   (e) $x^4 - 10x^2 + 9 = 0$ (**HINT**: Factor as $(x^2 - a)(x^2 - b) = 0$, and then solve for *x*.)

[†] This activity is referenced in Activity 1.5.

2. On one cold winter's morning, the temperature in Alfred between midnight and 4 a.m. can be modeled by the function $T(t) = 2t^2 - 7t + 5$, where T is in degrees Fahrenheit and t is hours after midnight.

(a) What was the temperature at midnight? (**HINT**: *T*-intercept)

(b) Find the times at which the temperature was 0°F. (**HINT**: *t*-intercepts)

(c) Determine the rate of change function for the temperature. Then compute how fast the temperature was changing at 3 a.m. Was the temperature rising or falling at 3 a.m.?

(d) At what time did the low temperature occur? What was the low temperature on this particular morning? (**HINT**: vertex)

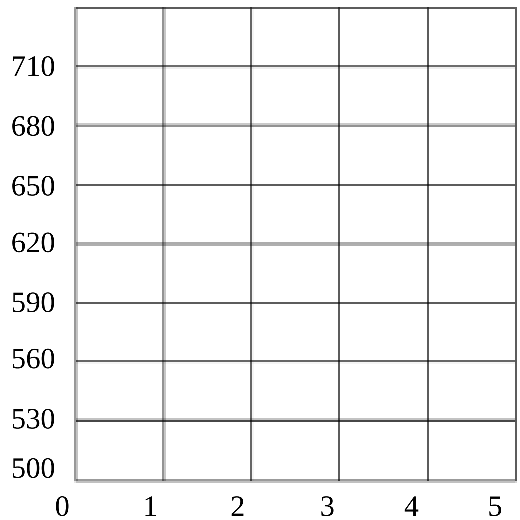
(e) Determine the interval on which the temperature was falling and the interval on which the temperature was rising. Show a number line with a sign test for *T'*, and write your answers using interval notation.

3. In Activity 1.5, an 800-gallon-capacity tank containing $V(t)$ gallons of water was being filled at a rate of $r(t)$ gallons per minute. Recall, the gauge that measures the volume of water in the tank malfunctioned, but we were able to find the volume in the tank at time t using integration. This data is shown in the table. We also found models for the volume and its rate of change as functions of t minutes after the malfunction:

$$r(t) = -30t + 110 \text{ gal/min}$$
$$V(t) = -15t^2 + 110t + 500 \text{ gal}$$

t (min)	$r(t)$ (gal/min)	$V(t)$ (gal)
0	110	500
1	80	595
2	50	660
3	20	695
4	–10	700
5	–40	675

(a) Sketch the graph of V by plotting points on the axes provided.



(b) Find the maximum volume in the tank by finding the vertex of the volume graph.

(c) How long did it take for the tank to empty completely? (i.e., when was the volume zero?)

4. Given the cost $C(x)$ of producing x items, the **marginal cost** (MC) is an approximation of the increase in cost of producing one more item by using the slope of the tangent line at production level x. With a "run" of 1 more item, the rise is simply the rate of change (see the figure below), so the marginal cost at a production level of x items is $C'(x)$.

The cost of producing x units of stuffed dog toys is $C(x) = 0.0002x^2 + 7x + 8000$ dollars. Find the marginal cost at a production level of 1000 items.