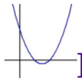
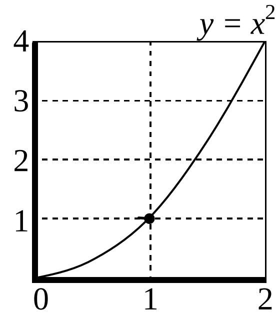


Examples 2.1 – Derivatives of Quadratic Functions

1. For the function $f(x)=x^2$, calculate the average rates of change for points closer and closer to and on either side of x_0 = 1. Guess $f'(1)$.



Solution: The average rate of change between (1, 1) and the arbitrary point (x, x^2) is $\frac{\Delta y}{\Delta x}=\frac{f(x)-f(x_0)}{x-x_0}=\frac{x^2-1}{x-1}$. Note that we cannot simply plug in x = 1 since the denominator is zero there. We use the formula to fill in the table with average rates of change between (1, 1) and points closer and closer to and on either side of x_0 = 1:

x	0.9	0.99	0.999	→	x_0 = 1	←	1.001	1.01	1.1
$\Delta y/\Delta x$	1.9	1.99	1.999	→	$f'(1)$ = ?	←	2.001	2.01	2.1

Finally, since the average rates of change are getting closer and closer to 2 from either side of $x_0=1$, we can safely guess that $f'(1)=2$.

2. Verify the guess in Part 1 by using the formula for the derivative of a quadratic.

Solution: From Lesson 2.1, we know that the derivative of $y=ax^2+bx+c$ is $y'=2ax+b$. In the case $f(x)=x^2$, we have a = 1 and b = c = 0. Therefore, $f'(x)=2x$, and $f'(1)=2$.

3. The time it takes an average athlete to swim 100 meters freestyle at age x years can be modeled by $T(x)=0.181x^2-8.463x+147.376$ seconds.

(a) Find the rates of change for a 13-year-old and a 25-year-old swimmer.
(b) At what age is the swim time the least? What is the swim time at that age?

Solution: (a) If $T(x)=0.181x^2-8.463x+147.376$ seconds, then $T'(x)=0.362x-8.463$ seconds per year of age. Therefore, the rate of change for a 13-year-old is $T'(13)=-3.757$ seconds per year. This means that swim time is decreasing. The rate of change for a 25-year-old is $T'(25)=0.587$ seconds per year, which means that swim time is increasing.

(b) By looking at the graph of T, we can see that swim time will be the least at the vertex of the parabola. Since the slope of the tangent line is zero there, set $T'(x)=0$ and solve for x:

$$0.362x-8.463=0 \quad\rightarrow\quad x=23.378$$

Therefore, the swim time will be the least at 23.378 years of age, and it is $T(23.378)=48.52$ seconds. In general, the x-intercepts of the derivative function tell where the original function has horizontal tangent lines.