## Examples 2.1 - Derivatives of Quadratic Functions

1. For the function $f(x)=x^{2}$, calculate the average rates of change for points closer and closer to and on either side of $x_{0}=1$. Guess $f^{\prime}(1)$.

## Solution:



| $x$ | 0.9 | 0.99 | 0.999 | $\rightarrow$ | $x_{0}=1$ | $\leftarrow$ | 1.001 | 1.01 | 1.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta y / \Delta x$ |  |  |  | $\rightarrow$ | $f^{\prime}(1)=$ | $\leftarrow$ |  |  |  |

2. Verify the guess in Part 1 by using the formula for the derivative of a quadratic.

## Solution:

3. The time it takes an average athlete to swim 100 meters freestyle at age $x$ years can be modeled by $T(x)=0.181 x^{2}-8.463 x+147.376$ seconds.
(a) Find the rates of change for a 13 -year-old and a 25 -year-old swimmer.
(b) At what age is the swim time the least? What is the swim time at that age?

Solution: (a)
(b)

Note: In general, the $x$-intercepts of the derivative function tell where the original function has horizontal tangent lines.

