Activity 2.1[†‡] – Derivatives of Quadratic Functions

FOR DISCUSSION: *Informally, what is the tangent line to a graph at a point?*
Describe the 3-step method for finding the slope of a tangent line.

1. A ball is thrown straight up into the air with an initial velocity of 85 ft/s. It's height in feet after t seconds is given by $h(t) = 85t - 16t^2$. Find the average velocity of the ball on the time intervals given in Parts (a) through (c).

    (**HINT**: Enter $85X - 16X^2$ into your calculator as Y1 and enter $(Y1(X) - Y1(2))/(X-2)$ into your calculator as Y2. Then use the table feature with Y2 and the right-hand endpoints.)

    (a) [2, 2.1]

    (b) [2, 2.01]

    (c) [2, 2.001]

    (d) Based on the above results, guess the instantaneous velocity when $t = 2$.

    $$v(2) \approx \underline{\hspace{3cm}}$$

    (e) Use the derivative formula for a quadratic to compute $v(t)$, and then check your answer to Part (d) by computing the exact value of $v(2)$.

    (f) Compute the acceleration function, $a(t)$.

[†] This activity is referenced in Lesson 2.5.
[‡] This activity has supplemental exercises.

2. Compute the first and second derivatives of each function using the differentiation formulas for quadratic and linear functions.. Be sure to use prime notation, as shown in Part (a).

(a) $f(x) = 2x^2 - 10x + 82$ $f'(x) =$ _____

                                                     $f''(x) =$ _____

(b) $y(w) = 2.5 - w + \frac{1}{2}w^2$

(c) $h(r) = (4r - 3)^2$ (**HINT**: Rewrite h as a quadratic function.)

3. The population of a bacterial colony after t hours is given by $P(t) = 4t^2 + 34t + 200$ bacteria. Find a formula for the growth rate, with units. Then determine how fast the population was growing after 2 hours.

4. The image to the right shows the graph of a quadratic function $y = f(x)$ along with the graph of the tangent line to the graph of f at $x = 4$.

(a) The best estimate of $f(4)$ is _____ .

(b) The best estimate of $f'(4)$ is _____ .

(c) Find a point-slope form of the tangent line at $x = 4$.
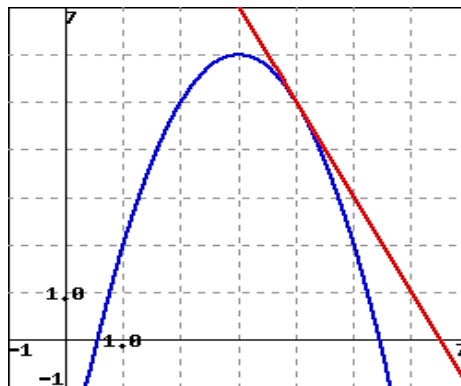
5. So far, we have used slopes of secant lines to approximate slopes of tangent lines. In this exercise we justify geometrically why this choice is a valid one for well-behaved functions.

   (a) On your calculator, graph the functions $Y_1 = -5x^2 + 7x + 8$ and $Y_2 = -3x + 13$ in the window $[0, 2] \times [8, 12]$. Note that Y_2 is the tangent line to the graph of Y_1 at $x = 1$.

   (b) "Zoom in" on the point $(1, 10)$ by pressing **[ZOOM] [2] [ENTER]**. Repeat this process two more times while visually observing what happens to the graph and the tangent line each time.

   Notice that the graph becomes indistinguishable from the tangent line near the point (1, 10) as you zoom in. In particular, secant lines near and from either side of (1, 10) have approximately the same slopes, and hence make good approximations of the tangent line.

   We say that the derivative "exists" (or that the function is "differentiable") at (1, 10) because the graph flattens out as we zoom in. Hence, the graph has a well-defined slope that we can estimate with as much accuracy as we wish.

6. In this exercise we investigate an example of how a derivative may *not* exist at a point.

   (a) On the TI-84, the **absolute value function** $y = |x| = \text{abs}(x)$ is entered as **[2ND]**, **[0]**, and **[ENTER]**. Turn off the graphs of Y_1 and Y_2, and then graph the function $Y_3 = 2 \cdot |1 - x| + 10$ in the window $[0, 2] \times [8, 12]$. Notice that the graph is not "smooth" (i.e., it has a corner). The point $(1, 10)$ will be at the center of your screen.

   (b) Zoom in on the point $(1, 10)$ by pressing **[ZOOM] [2] [ENTER]**. Repeat this process as many times as you wish while visually observing what happens to the graph each time.

   Notice that the graph never flattens out no matter how much you zoom in. In particular, notice that secant lines from the left and secant lines from the right will have very different slopes, namely, –1 and 1, respectively.

   We say that the derivative "does not exist" (or that the function is "not differentiable") at (1, 10) because there is not a single line with which to define the slope of the graph at (1, 10).