

Activity 1.5^{†‡} – Rectilinear Motion

FOR DISCUSSION: *Explain how position, velocity, speed, and acceleration are related.*

Explain displacement in both physical and mathematical terms.

Explain total distance traveled in both physical and mathematical terms.

1. The position of an object is given by $s(t) = 35 - t$ feet, where t is in seconds. Use differentiation to determine the velocity, speed, and acceleration functions of the object. Then compute the position, velocity, speed, and acceleration of the object at $t = 5$ seconds. Include **units** in all answers.

Position $s(t) = 35 - t$ feet

Position $s(5) =$

Velocity $v(t) =$

Velocity $v(5) =$

Speed $|v(t)| =$

Speed $|v(5)| =$

Acceleration $a(t) =$

Acceleration $a(5) =$

2. An object travels along a line with constant velocity -20 m/s and initial position 100 m. Include **units** in all answers.

(a) Determine the position function of the object.

(b) Determine the time at which the position of the object is zero.

(c) Use integration to determine the net change in the object's position (i.e., displacement) from time 9.5 s to time 12.5 s.

[†] This activity is referenced in Activity 2.2.

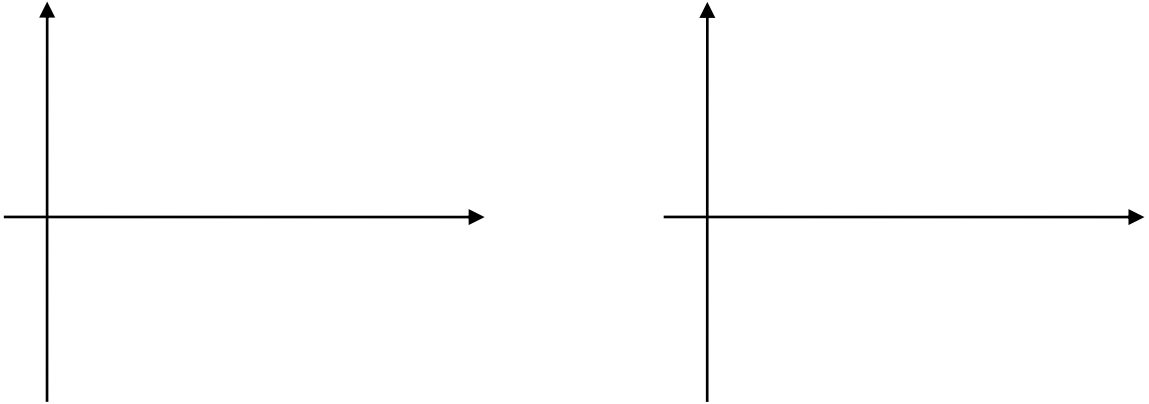
[‡] This activity has supplemental exercises.

- (d) Determine the total distance traveled over the same time period by integrating the speed function. (Note: Your answer should not contain an absolute value.)

(The displacement is not always the absolute value of the total distance traveled! See Problem 3 below.)

3. Suppose $v(t) = 10 - 4t$ is the velocity (in meters per second) of an object in rectilinear motion.

- (a) Carefully sketch the graphs of the velocity v and the speed $|v|$ on the time interval $[0, 4]$ on two separate sets of axes. (**HINT:** Absolute value reflects negative sections of a graph across the x -axis and leaves positive sections alone.)



- (b) The t -intercept of $v(t)$ is the time at which the velocity is zero. Determine the t -intercept.

- (c) Use the graph of $v(t) = 10 - 4t$ and **geometry** to find the displacement of the object on the time interval $[0, 4]$.

$$\int_0^4 (10 - 4t) dt =$$

- (d) Use the graph of $|v(t)| = |10 - 4t|$ and **geometry** to find the total distance traveled by the object on the time interval $[0, 4]$.

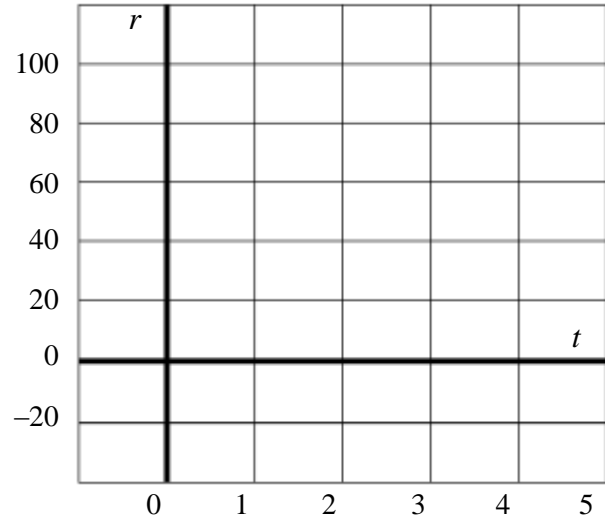
$$\int_0^4 |10 - 4t| dt =$$

4. Thus far, we have used geometry to find the definite integral of a linear function because we do not yet know its antiderivative. In this activity, we will discover a clue as to the form of the antiderivative of a linear function.

An 800-gallon-capacity holding tank containing $V(t)$ gallons of water was being filled at a rate of $r(t)$ gallons per minute. At exactly 8:00 a.m., the gauge that measures the volume of water in the tank malfunctioned. At that moment (at time $t = 0$), it read 500 gallons. Workers were able to measure how fast water was entering or leaving the tank, but not the volume of water in the tank. This data is shown in the table to the right. (Do not attempt to fill in the table just yet.)

t (min)	$r(t)$ (gal/min)	$V(t)$ (gal)
0	110	500
1	80	
2	50	
3	20	
4	-10	
5	-40	

- (a) How can you tell by looking at the data that $r(t)$ is linear? Write a linear model for $r(t)$, with units, and sketch the graph on the axes provided. (HINT: Slope-intercept form.)



- (b) Find the t -intercept of $r(t)$. That is, find the time at which the water stopped entering the tank. (Note that this is the time at which the tank's volume was at its maximum.)

- (c) The third column of the table above tells the total volume of water in the tank t minutes after the malfunction. Apply the concept of integration (as it relates to accumulated change) to fill in the table by using the graph of $r(t)$ and geometry. (HINT: $V(1)$ is 500 plus the area below r from 0 to 1, etc. Remember to subtract any areas below the t -axis.)

(d) Follow the instructions to enter the input and output data for the volume function $V(t)$ into your calculator and view a scatter plot of the volume.

- Press [STAT] and choose **1:Edit**, which will bring you to the list menu.
- Clear old data by moving the cursor to the heading of a column, press [CLEAR] [ENTER], and then move the cursor back down to the top of the list.
- Type inputs into L_1 , one at a time, pressing [ENTER] after each one. Repeat for the outputs into L_2 .
- To view a scatter plot, press [2ND] [Y=] to go to the STAT PLOTS menu.
- Choose **1:Plot1**, turn the plot **ON**, and press [ENTER]. You might want to turn off or clear any other equations.
- Now press [ZOOM] and choose **9:ZoomStat**.

(e) What type of function will best fit this data? That is, what type of function do you think is the integral of the linear function $r(t)$?

(f) Follow the instructions to find a quadratic regression model for the volume $V(t)$.

- Press [STAT] and then [▶] to the CALC menu. This will bring you to the list of regression models. Press [5] to choose the **QuadReg** option.
- Depending on your calculator model, do one of the following:
 - Press [2ND] [1] [,] [2ND] [2] [,] [VARS] [▶] [1] [1]. You should see **QuadReg L_1, L_2, Y_1** on the home screen, OR
 - Fill in Xlist (press [2ND] [1] [,]), Ylist (press [2ND] [2] [,]), and Store RegEQ (press [VARS] [▶] [1] [1]).
- Press [ENTER]. The model will be stored in the graph menu under Y_1 . Press [GRAPH] to view the graph of the model over the scatter plot.

Write down the model for the volume $V(t)$, with units. It is stored as Y_1 in the graph menu.

We will work with this example a little more in Lesson 2.2.