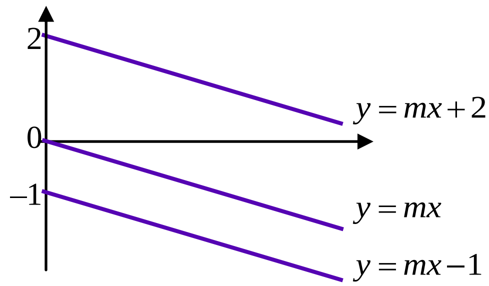
Lesson 1.4 – Integrals of Constant Functions

Given a linear function $y = mx + b$, we can find a unique constant slope (derivative) $y' = m$. However, given a constant function $y' = m$, there are an infinite number of lines having m as a slope. Lines that have the same slope m are vertical shifts of one another and have equations of the form $y = mx + C$ for some constant C. These lines form **the family of antiderivatives** of m, which is denoted by the **indefinite integral**:



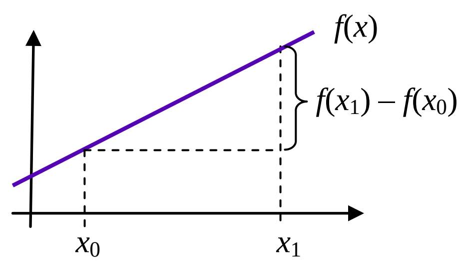
$$\int m\,dx = mx + C \quad \text{(family of antiderivatives of } m\text{)}$$

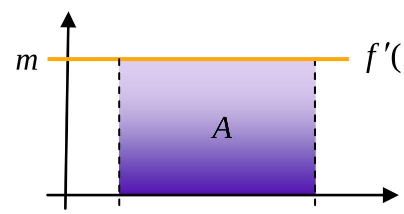
(The function between the **integral sign** $\int$ and the **differential** dx is called the **integrand**)

Suppose we are given the rate of change $f'(x)$ over some interval $[x_0, x_1]$ and we want the net (accumulated) change in f on $[x_0, x_1]$.

Observation 1: If f has a constant rate of change $f'(x) = m$ on $[x_0, x_1]$, then f is an antiderivative of m of the form $f(x) = mx + C$, for some constant C. Note that the net change in $f(x) = mx + C$ on $[x_0, x_1]$ for *any* C is

$$f(x_1) - f(x_0) = (mx_1 + C) - (mx_0 + C) = mx_1 - mx_0$$





Observation 2: Let A denote the net (signed) area bounded by f' from x_0 to x_1. From geometry,

$$A = m(x_1 - x_0) = mx_1 - mx_0 = f(x_1) - f(x_0)$$

The net (signed) area bounded by the constant function $y = m$ on the interval $[x_0, x_1]$ is denoted by the

Definite integral: $\int_{x_0}^{x_1} m\,dx$ = net signed area bounded by m on $[x_0, x_1]$

(Units: (units of m) × (units of x))

Fundamental Theorem of Calculus (for constant functions): If $f'(x)$ is a constant function, then the net area bounded by the graph of $f'(x)$ on the interval $[x_0, x_1]$ is equal to the change in the (or any) antiderivative $f(x)$ on $[x_0, x_1]$. That is,

$$\int_{x_0}^{x_1} f'(x)dx = f(x)\Big|_{x_0}^{x_1} = f(x_1) - f(x_0)$$