Examples 1.4 – Integrals of Constant Functions

1. Evaluate each of the following.

    (a) $(9x - 12)'$ (b) $\int 9\, dt$ (c) $(2.991 - 0.423u)'$ (d) $\int (-0.423)\, dT$

    Solution: (a) The slope of the linear function $y = 9x - 12$ is $(9x - 12)' = 9$.

    (b) The family of linear functions that have slope $m = 9$ is $\int 9\, dt = 9t + C$.

    (c) The slope of the linear function $y = 2.991 - 0.423u$ is $(2.991 - 0.423u)' = -0.423$.

    (d) The family of functions that have slope $m = -0.423$ is $\int (-0.423)\, dT = -0.423T + C$.

2. If a savings account increases by \$110 per month, then how much money is saved from month 5 to month 10?

    Solution: We must compute the net change in the amount in the account over [5, 10] given that the rate of increase is \$110 per month. To do so, we need a member of the family

    $$\int 110\, dt = 110t + C$$

    so we simply choose the one with $C = 0$. Therefore, by the Fundamental Theorem, we have

    $$\int_5^{10} 110\, dt = \left(110t\right)\Big|_5^{10} = 110(10) - 110(5) = 550 \text{ dollars.}$$

    (We could have computed the answer algebraically by multiplying \$110 by 5 months, but only because the rate is constant. We will see that the FTC holds for variable rates as well.)

3. Evaluate $\int_{-3}^{6} (-4)\, dx$ and sketch the geometrical interpretation of the answer.

    Solution: By the Fundamental Theorem, $\int_{-3}^{6} (-4)\, dx = (-4x)\Big|_{-3}^{6} = (-4)(6) - (-4)(-3) = -36$,

    which represents the net signed area between the graph of $y = -4$ and the x-axis on [–3, 6]: