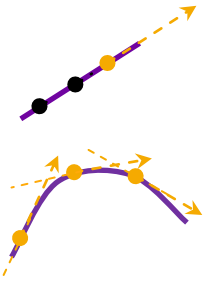
Lesson 1.3 – Derivatives of Linear Functions

Suppose A is the altitude function of a hiking trail. If A is linear, then the trail has a constant steepness, no matter where we are standing. We measure the **steepness of the line** at a given point by its **slope**. If A is nonlinear, then the steepness of the trail varies, depending on where we are standing. We measure the **steepness of the curve** at a given point by the **slope of the tangent line**. Informally, a **tangent line** to a curve is a line that intersects the curve at a point and points in the same direction as the curve does at that point. The slope of the tangent line at a point, if one exists, is better known as the **derivative**.

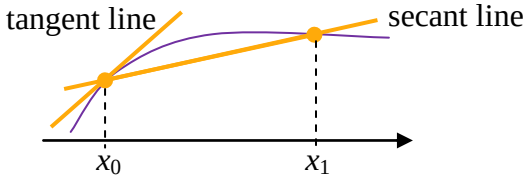


Precalculus question: How do we measure the change in a function over an interval?

1. Average rate of change on an interval.
2. Slope of the line between two points.
3. Slope of the secant line between two points.

Calculus question: How do we measure the change in a function at a single point?

1. (Instantaneous) rate of change at a point.
2. Slope of the curve at a point.
3. Slope of the tangent line at a point.
4. Derivative at a point.



The **derivative of a function at a point** (in the domain) is the slope or rate of change of the curve at that point, if such a number exists. (We will discuss existence later.)

Prime notation: $f'(x_0)$ = the derivative of f at a given x_0 (slope of f at a given x_0)

The **derivative function** assigns to each function input the slope of the curve at that input if one exists. (We will discuss existence later.) This yields a new function.

Prime notation: $f'(x)$ = the derivative function of f (slope of f at any x).

The **second derivative function** is the derivative of the derivative function. This function tells how fast the rate of change is changing.

Prime notation: $f''(x)$ = the second derivative function of f (slope of f' at any x).

Derivatives of linear and constant functions: The slope at a point on a line is equal to the slope between any two points. Therefore, the derivative of a linear function at any point is its slope:

If $f(x) = mx + b$, then $f'(x) = m$. In particular, if $f(x) = b$, then $f'(x) = 0$.