Examples 1.3 – Derivatives of Linear Functions

1. Find the first and second derivatives of $y = 4x + 1$, $g(t) = 3 – 5t$, and $h(r) = 1.344$.

   Solution: Since all three of the given functions are linear, the derivative of each function is simply its slope. That is, $y' = 4$, $g'(t) = –5$, and $h'(r) = 0$. For the same reason, the second derivatives are $y'' = 0$, $g''(t) = 0$, and $h''(r) = 0$.

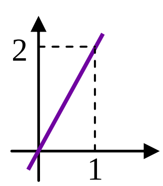
2. The rate of change of the position over time of a moving object is its **velocity** $v(t)$, and the rate of change of velocity over time is its **acceleration** $a(t)$. If the position of an object after t minutes is given by $s(t) = 65t + 20$ cm, then what are its velocity and acceleration functions?

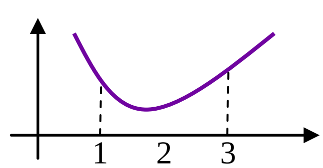
   Solution: In general, if $s(t) = 65t + 20$, then $s'(t) = 65$ and $s''(t) = 0$. We must express our answers in the context of the problem with appropriate units, and the words "over time" give us a hint as to how to do this: If $s(t) = 65t + 20$ cm, then the velocity $v(t) = s'(t) = 65$ cm/min (centimeters per minute). If $v(t) = s'(t) = 65$ cm per minute, then the acceleration function is $a(t) = v'(t) = s''(t) = 0$ cm per minute per minute, or cm/min^2.

4. For each part, sketch an example of a (possibly nonlinear) graph having the given properties.

   (i) A constant derivative of two.
   (ii) A negative derivative at $x = 1$, and a positive derivative at $x = 3$.
   (iii) A zero derivative at $x = –1$, positive derivatives on the interval $(–1, 2)$, and a zero derivative at $x = 2$.

   Solution: There are many correct solutions. Here are some possibilities.

(i)


(ii)


(iii)