Activity 1.3‡ – Derivatives of Linear Functions

FOR DISCUSSION: *Give three interpretations of the word* derivative.
Informally, what is a tangent line?
What is the derivative of the function $y = mx + b$*? What about* $y = b$*?*
If $h'(x) = 5$ *for all* x*, then what can you say about* h*?*

---

1. Suppose m and b are real constants. Compute the derivative (slope) of each function. Write your answer using **prime notation**. That is, if $f(x)$ is given, then $f'(x)$ denotes the derivative.

(a) $f(x) = b$

(b) $g(x) = 6$

(c) $h(x) = -1.5$

(d) $F(x) = mx + b$

(e) $G(x) = 9x - 7$

(f) $H(x) = -x + 2$

---

‡ This activity has supplemental exercises.

2. The position of an object is given by $s(t) = -2t + 5$ feet (ft), where t is in seconds (s). Determine each of the following, and include proper notation and units.

   (a) The velocity function of the object.

   (b) The velocity after 10 seconds.

   (c) The acceleration function of the object. (Write ft/s^2 instead of ft/s/s.)

3. The function $H(t)$ measures the amount of helium in a tank, in cubic feet, at time t hours. Suppose $H(t)$ is a linear function such that $H(1) = 22$ ft^3 and $H'(1) = -0.4$ ft^3/hr.

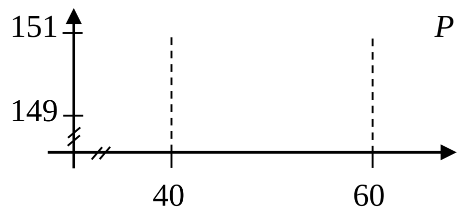
   (a) Determine a formula for H. (**HINT**: First find the point-slope form.)

   (b) Compute the amount of helium in the tank after 5 hours.

4. Let $P(x)$ be the profit in dollars that the AU Math Club makes selling x number of t-shirts. Suppose that $P(50) = 150$ dollars and $P'(50) = -0.1$ dollars per shirt.

(a) Suppose that when the Math Club sells between 40 and 60 t-shirts, the profit function P is a linear function of the form $P(x) = ax + b$. Using the information given above, find a formula for P, with units.

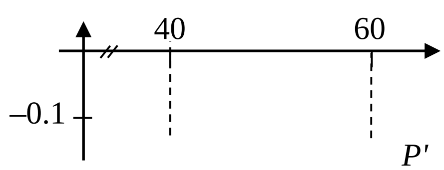
(b) Sketch the graph of the formula for P on the interval [40, 60].



(c) Compute the net change in P on the interval [40, 60].

(d) Find a formula for P' with units.

(e) Sketch the graph of the formula for P' on the interval [40, 60].



(f) Compute the "net" area of the rectangle bounded by P' and the x-axis on [40, 60]. That is, since the rectangle lies below the x-axis, use -0.1 as the "height" of the rectangle.

(g) What do you notice about your answers to Parts (c) and (f)?

(*We will see that the net change in a function on an interval is the same as the net area bounded by its derivative graph on that interval. This is a "fundamental theorem."*)