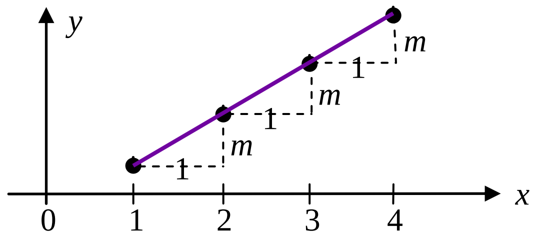
Lesson 1.2 – Linear Functions

A **linear function** is a rule for which each unit change in input produces a constant change in output. The constant change is called the **slope** and is usually denoted by *m*.



Slope formula: If (x_1, y_1) and (x_2, y_2) are *any* two distinct points on a line, then the slope is

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

(An average rate of change!)

Equations for lines:

Slope-intercept form: $y = mx + b$ — m is the **slope** of the line; b is the ***y*-intercept** (i.e., **initial value**, $y(0)$).

Point-slope form: $y - y_0 = m(x - x_0)$ — m is the **slope** of the line; (x_0, y_0) is any **point** on the line.

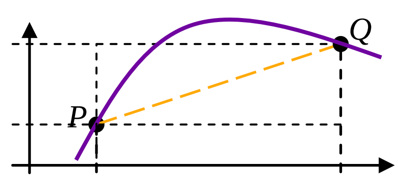
Domain: All real numbers.

Graph: A line with no breaks, jumps, or holes. (A graph with no breaks, jumps, or holes is said to be **continuous**. We will formally define *continuity* later in the course.)

A **constant function** is a linear function with slope $m = 0$. The graph of a constant function is a **horizontal line**, and its equation has the form $y = b$. A **vertical line** has equation $x = a$, but this is not a function since it fails the vertical line test.

Notes:

1. A positive slope means the line is increasing, and a negative slope means it is decreasing.
2. If we walk from left to right along a line passing through distinct points *P* and *Q*, then we will experience a constant steepness equal to the slope of the line. If we stand still at *P*, *Q*, or any other point on the line, then we will experience the same steepness.
3. If $y = f(x)$ is *any* function defined at distinct points *P* and *Q*, then the average rate of change from *P* to *Q* is the slope of the **secant line** between *P* and *Q*.



4. The slope of the secant line between two points describes the relationship between initial and final values, but not intermediate behavior.