Examples 1.2 – Linear Functions

1. Find two forms of the equation of the line with slope –4 passing through the point (3, 2).

   Solution: Since we are given a point and a slope, we can use the point-slope form to get

   $$y - 2 = -4(x - 3)$$

   Solving for y in the point-slope form gives the slope-intercept form

   $$y = -4x + 14$$

2. Find two forms of the equation of the line passing through the points (–2, 1) and (1, 2).

   Solution: The slope of the line through the given points is $m = \frac{\Delta y}{\Delta x} = \frac{2-1}{1-(-2)} = \frac{1}{3}$.

   Therefore, two point-slope forms are

   $$y - 1 = \tfrac{1}{3}(x - (-2)) \quad \text{and} \quad y - 2 = \tfrac{1}{3}(x - 1)$$

   Solving for y in either point-slope form gives the slope-intercept form

   $$y = \tfrac{1}{3}x + \tfrac{5}{3}$$

   Note that the slope-intercept form of a line is unique, while there are infinitely many point-slope forms.

3. Find an equation of the line passing through the points (0, 4) and (–3, 4).

   Solution: The slope of the line through the given points is $m = \frac{\Delta y}{\Delta x} = \frac{4-4}{-3-0} = \frac{0}{-3} = 0$.

   Since the slope is zero and the y-intercept is $b = 4$, the line is the constant function $y = 4$.

4. Find an equation for the line passing through the points given in Examples 1.1.

   Solution: In Examples 1.1, we found that the slope is –4.5, and we are given a point (2, 80). The point-slope and slope-intercept forms are, respectively,

   $$y - 80 = -4.5(x - 2) \quad \text{and} \quad y = -4.5x + 89$$

   We could have also used the point (6, 62) to get $y - 62 = -4.5(x - 6)$.