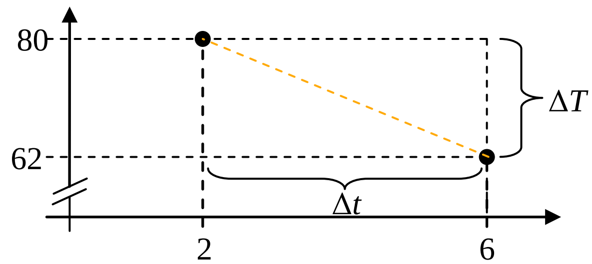
Examples 1.1 – Average Rate of Change

Let $T(t)$ be the air temperature in degrees Fahrenheit between 6 A.M. and 8 P.M. where t is hours after noon. Suppose $T(2)=80$ and $T(6)=62$.

1. Sketch a graph of the given data points and mark and label the net changes ΔT and Δt.

   Solution:



2. Describe the meanings of the expressions $T(2)=80$ and $T(6)=62$ in words with units.

   Solution: The expression $T(2)=80$ means that at 2 hours after noon, or at 2:00 P.M., the temperature was 80°F. The expression $T(6)=62$ means that at 6 hours after noon, or at 6:00 P.M., the temperature was 62°F.

3. Find ΔT and $\Delta T/\Delta t$ on the interval [2, 6], and describe the answers in words with units.

   Solution: We have

$$\Delta T = T(6) - T(2) = 62 - 80 = -18$$

   This means that between 2:00 P.M. and 6:00 P.M, the temperature decreased by 18°F.

   Furthermore,

$$\frac{\Delta T}{\Delta t} = \frac{T(6)-T(2)}{6-2} = \frac{62-80}{6-2} = \frac{-18}{4} = -4.5$$

   which means that between 2:00 P.M. and 6:00 P.M, the temperature decreased by 4.5°F per hour, on average.

4. What can be said about the temperature at 4:00 P.M.?

   Solution: Change and average rate of change do not describe intermediate behavior, so the best we could do is estimate the temperature at 4:00 P.M. using the average rate of change. At 2:00 P.M., the temperature was 80°F, and the average rate of change was –4.5°F. Therefore, the temperature at 4:00 P.M. was approximately (–4.5)(2) + 80 = 71°F.

5.