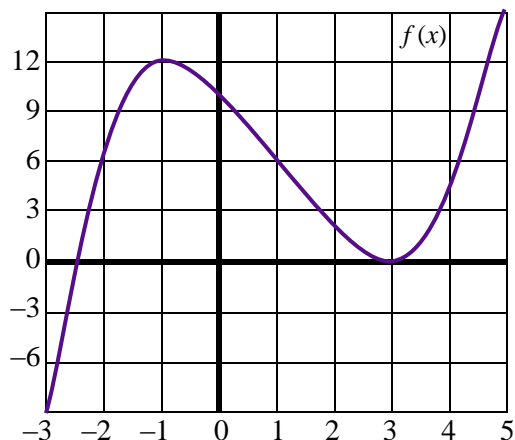


Activity 1.1 – Average Rate of Change

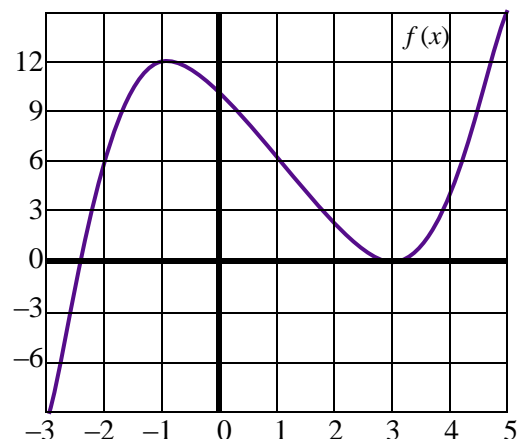
FOR DISCUSSION: *In your own words, describe the major aspects of the graph shown below.*

1. Suppose the function $y = f(x)$ represents a town's daily high temperature in degrees Celsius, where x is the number of days since New Year's Day, $-3 < x < 5$ ($x = 0$ corresponds to January 1).



- (a) On which interval(s) was the temperature above zero (positive)? On which interval(s) was the temperature equal to or below zero (negative)? Use interval notation to express your answers. That is, express time intervals using parentheses or brackets.
- (b) On which interval(s) do you think the daily temperature was rising (increasing)? Use interval notation to express your answers. Do you think the average rate of change of f on these intervals is positive or negative?
- (c) On which interval(s) do you think the daily temperature was falling (decreasing)? Use interval notation to express your answers. Do you think the average rate of change of f on these intervals is positive or negative?

- (d) Does the graph have peaks or valleys? Where, and what do they mean in this context?



- (e) An x - or y -intercept is a point at which the graph touches or crosses the x - or y -axis, respectively. Estimate the x - and y -intercepts and describe their meaning in this context.

- (f) For each average rate of change below, find its value, describe its meaning in context with units, and then connect the corresponding points on the graph with a line segment.

(i) $\frac{f(3) - f(0)}{3 - 0} =$

(ii) $\frac{f(-1) - f(-2)}{(-1) - (-2)} =$

2. The table shows the average temperature T at depth d inside a borehole.

Depth d in meters (m)	Temperature T in degree Celsius ($^{\circ}\text{C}$)
25	5.50
50	5.20
75	5.10
100	5.10
125	5.30
150	5.50
175	5.75
200	6.00

(a) Compute the following, with units:

$$\frac{T(125) - T(25)}{125 - 25} =$$

(b) Compute the following on the interval $[50, 125]$, with units.

$$\frac{\Delta T}{\Delta d} =$$

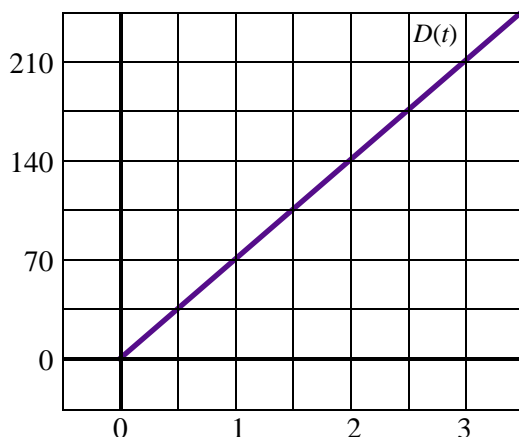
(c) Find the average rate of change of T on the interval $[125, 200]$, with units.

(d) Choose all statements below that give a correct interpretation of the answer to Part (a).

There may be more than one correct answer. (HINT: Pay close attention to units.)

- (i) The temperature changes by a total of -0.002 degrees Celsius when moving from a depth of 25 meters to 125 meters.
- (ii) On average, the temperature decreases at a rate of 0.002 degrees Celsius per minute over the interval $[25, 125]$.
- (iii) Over the interval from 25 m to 125 m, the temperature decreases on average at a rate of 0.002 degrees Celsius per meter.
- (iv) The temperature is changing at a rate of -0.002 degrees Celsius per minute when the depth is 25 meters.
- (v) The temperature at 26 meters is approximately 5.498 degrees Celsius.

3. The graph shows the distance D , in miles (mi), traveled by a vehicle as a function of time t , in hours (hr).



(a) How would you best describe the “shape” of the graph?

(b) For each time interval in the table below, find the values of ΔD , Δt , and $\frac{\Delta D}{\Delta t}$.

Time Interval	ΔD	Δt	$\frac{\Delta D}{\Delta t}$
[1, 2.5]			
[0, 2]			
[0.5, 3]			

(c) Based on your results from Part (b), the average rate of change of D is constant, and it does not depend on which interval of time we chose. What is the constant rate of change of D ? Write your answer using correct notation and units.

(d) In physical terms, what does the answer to Part (c) tell us about the motion of the vehicle?

Observe that a line has a constant average rate of change. This constant is used to measure the steepness, or “slope,” of the line. In other words, if we walk from left to right along a line, then we will experience a constant steepness equal to the slope of the line. Moreover, if we stand still at any point on the line, then we will experience the same steepness.